**Summary of Boundary Resistance**

Electrical: Work function ($\Phi$) mismatch

Thermal: Debye ($v, \Theta$) mismatch

**Acoustic vs. Diffuse Mismatch Model**

Acoustic Impedance Mismatch (AIM)  

$$= \frac{(\rho v)_1}{(\rho v)_2}$$

**Acoustic Mismatch Model (AMM)**  
Khalatnikov (1952)

Diffuse Mismatch Model (DMM)  
Swartz and Pohl (1989)

Snell’s law with $Z = \rho v$

$$T = \frac{4Z_1Z_2}{(Z_1 + Z_2)^2}$$  
(normal incidence)
Debye Temperature Mismatch

<table>
<thead>
<tr>
<th>Material</th>
<th>( \theta_\text{D} ) (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Au</td>
<td>105</td>
</tr>
<tr>
<td>Pt</td>
<td>240</td>
</tr>
<tr>
<td>Al</td>
<td>428</td>
</tr>
<tr>
<td>Cr</td>
<td>630</td>
</tr>
<tr>
<td>GaN</td>
<td>695</td>
</tr>
<tr>
<td>Si</td>
<td>645</td>
</tr>
<tr>
<td>Sapphire</td>
<td>1035</td>
</tr>
<tr>
<td>AlN</td>
<td>1150</td>
</tr>
</tbody>
</table>

Stevens, J. Heat Transf. 127, 315 (2005)

General Approach to Boundary Resistance

(conductance)

Flux \( J = \#\text{incident particles} \times \text{velocity} \times \text{transmission prob.} \)

\[
G_{B,e} = R_{B,e}^{-1} = \frac{dJ_e}{dV} \\
G_{B,th} = R_{B,th}^{-1} = \frac{dJ_{th}}{dT}
\]

\[
J = J_{A\rightarrow B} - J_{B\rightarrow A} = (\#\text{incident}) \int g_A g_B (f_A - f_B) T(E) dE
\]

More generally:
- fancier version of Landauer formula!
- ex: electron tunneling

\[
T_{\text{WKB}} \approx \exp \left( -2 \int |k(x)| dx \right)
\]
### Band-to-Band Tunneling Conduction

- Assuming parabolic energy dispersion $E(k) = \frac{\hbar^2 k^2}{2m^*}$

\[
T(E_c) \approx \exp \left( -\frac{4\sqrt{2m^* E_c^{3/2}}}{3q\hbar F} \right)
\]

$F =$ electric field

- E.g. band-to-band (Zener) tunneling in silicon diode

\[
J_{BB} = \frac{q^3 F V_{eff}}{4\pi^3 \hbar^2} \sqrt{\frac{2m^*}{E_G}} \exp \left( -\frac{4\sqrt{2m^* E_G^{3/2}}}{3q\hbar F} \right)
\]

See, e.g. Kane, J. Appl. Phys. 32, 83 (1961)

### Thermionic and Field Emission (3D)

- Thermionic emission
- Tunneling (field emission)

\[
J_{TE} \approx \frac{4\pi m^* k_B^2 q}{h^3} T^2 \exp \left( -\frac{q\Phi}{k_B T} \right)
\]

\[
J_{FN} \approx \frac{q^3 F^2}{8\pi h\Phi} \exp \left( -\frac{4\sqrt{2m^* \Phi^{3/2}}}{3q\hbar F} \right)
\]

Field emission a.k.a. Fowler-Nordheim tunneling

**The Photon Radiation Limit**

Phonons behave like photons at low-T in the absence of scattering (why?)

Acoustic analog of Stefan-Boltzmann constant

\[ \sigma_{\text{photon}} = \frac{\pi^4 k_B^4}{120 T^4} \sum \frac{1}{c_i^2} \]

\( c_i = \) sound velocities (2 TA, 1 LA)

Heat flux:

\[ \dot{Q} = \sigma A (T_2^4 - T_1^4) \]

what’s the temperature profile?

Swartz & Pohl, Rev. Mod. Phys. 61, 605 (1989)

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**Phonon Conductance of Nanoconstrictions**


1) \( a \gg \lambda \rightarrow \tau = \cos(\theta) \)

\[ G_p = R_{K_p} = \frac{A}{4} \sum \int C(\omega) v(\omega) d\omega \]

\[ G_p = \frac{A \pi^2}{30} (k_B^2/\hbar^2) (v_L^{-2} + 2v_T^{-2}) T^3 \]

\( \lambda = \) dominant phonon wavelength
Phonon Conductance of Nanoconstrictions

2) $a \ll \lambda$

$$\tau_{eff} = \tau_0 \tau_R (\tau_k + \tau_R)$$

$$\tau_R = \left(\frac{8\pi^2}{\omega a v}\right)^4 \cos^2 \theta = \left(\frac{8\pi^2}{\omega a v}\right)^4 \cos^2 \theta$$

$$G_{h,R} = \frac{128}{1215} \pi^3 a^8 (k_B^2 / h^2) (v_L^2 + 2 v_T^2) T^3$$

$\lambda = \text{dominant phonon wavelength}$


Why Do Thermal Boundaries Matter?

- Because surface area to volume ratio is greater for nanowires, nanoparticles, nanoconstrictions
- Because we can engineer metamaterials with much lower “effective” thermal conductivity than Mother Nature
Because of Thermoelectric Applications

- No moving parts: quiet and reliable
- No Freon: clean

Thermoelectric Figure of Merit (ZT)

Coefficient of Performance

\[ \text{COP}_{\text{max}} = \frac{T_c}{T_h - T_c} \sqrt{\frac{1 + z T_m}{1 + z T_m + 1}} \]

where ZT is...

Seebeck coefficient

Electrical conductivity

\[ ZT \equiv \frac{S^2 \sigma}{\kappa} \]

Temperature

Thermal conductivity

\( T_c = 300 \text{ K} \)

\( T_h = 250 \text{ K} \)

Bi\(_2\)Te\(_3\) Freon (CFCs)

Courtesy: L. Shi
ZT State of the Art

- Goal: decrease $k$ (keeping $\sigma$ same) with superlattices, nanowires or other nanostructures
- Nanoscale thermal engineering!

Harman et al., Science 297, 2229

Venkatasubramanian et al. Nature 413, 597
Nanoscale Thermometry

Typical Measurement Approach

• For nanoscale electrical we can still measure \( \frac{I}{\Delta V} = \frac{\Delta J_q}{\Delta V} \)

• For nanoscale thermal we need \( \frac{J_{th}}{\Delta T} \), but there is NO good, reliable nanoscale thermometer

• Typically we either:
  a) Measure optical reflectivity change with \( \Delta T \)
  b) Measure electrical resistivity change with \( \Delta T \) (after making sure they are both calibrated)

• Must know the thermal flux \( J_{th} \)
The 3ω Method (thin film cross-plane)

- $I \sim 1\omega$
- $T \sim T^2 \sim 2\omega$
- $R \sim T \sim 2\omega$
- $V_{3\omega} \sim IR \sim 3\omega$

$$\Delta T(\omega) = \frac{2V_{3\omega}}{\alpha T_0 R_0}$$

where $\alpha$ is metal line TCR

$\Delta T(2\omega) = \frac{P}{Lb} \left[ \frac{1}{2} \ln \left( \frac{b^2}{a^2} \right) + \frac{\eta}{2} \ln(2\omega) - \frac{i\pi}{4} \right] + \frac{Pd}{2Lbk_1}$

D. Cahill, Rev. Sci. Instrum. 61, 802 (1990)

3ω Method Applications…


Compare temperature rise of metal line for different line widths, deduce anisotropic polymer thermal conductivity

Thin crystalline Si films

The 3ω Method (longitudinal)

Lu, Yi, Zhang, Rev. Sci. Instrum. 72, 2996 (2001)

- Low frequency: $V(3ω) \sim 1/k$
- High frequency: $V(3ω) \sim 1/C$
- Tested for a 20 mm diameter Pt wire
- Results for a bundle of MW nanotubes:
  $C \sim \text{linear } T \text{ dependence, low } k \sim 100 \text{ W/mK}$

- 3ω mechanism: $ΔT \sim V^2/k$ and $R \sim R_o + αΔT$

Another Suspended Bridge Approach

Source: L. Shi
**Measurement Scheme**

Thermal Conductance:

\[ G_t = \frac{Q_H + Q_L}{T_h + T_s - 2T_0} - \frac{T_s - T_0}{T_h - T_s} \]

\[ Q_H = IR_H \]

\[ Q_L = IR_L \]

Thermopower:

\[ Q = V_{TE} \]

\[ G_t = kA/L \]

**Multiwall Nanotube Measurement**


Resistance vs. Joule Heat

\[ R = \frac{\Delta R}{R} \]

\[ R_2 = \frac{R_1}{R_0} \]

Measurement result

\[ k \propto T^2 \]

\[ l \approx 0.5 \mu m \]

Cryostat: T : 4-350 K

\[ P \approx 10^{-6} \text{ torr} \]
### Scanning Thermal Microscopy (SThM)

- Sharp temperature-sensing tip mounted on cantilever
- Scan in lateral direction, monitor cantilever deflection
- Thermal transport at tip is key (air, liquid, and solid conduction)


### SThM Applied to Multi-Wall Nanotube

- Must understand sample-tip heat transfer
- Note arbitrary temperature units here (calibration was not possible)
- Note $R_{\text{tip}} \sim 50$ nm vs. $d \sim 10$ nm

Source: L. Shi, D. Cahill
Scanning Joule Expansion Microscopy

- AFM cantilever follows the thermo-mechanical expansion of periodically heated ($\omega$) substrate
- Ex: SJEM thermometry images of metal interconnects
- Resolution $\sim$10 nm and $\sim$degree C

Source: W. P. King

Thermal Effects on Devices

- At high temperature ($T \uparrow$)
  - Threshold voltage $V_t \downarrow$ (current $\uparrow$) $V_t = V_{t_0} + \eta(T - T_0)$
  - Mobility decrease $\mu \downarrow$ (current $\downarrow$) $\mu = \mu_0 (T / T_0)^\alpha$
  - Device reliability concerns
- Device heats up during characterization (DC I-V)
  - Temperature varies during digital and analog operation
  - Hence, measured DC I-V is not “true” I-V during operation
  - True whenever $\tau_{\text{thermal}} >> \tau_{\text{electrical}}$ and high enough power
  - True for SOI-FET (perhaps soon bulk-FET, CNT-FET, NW-FET)
- How to measure device thermal parameters at the same time as electrical ones?
Measuring Device Thermal Resistance

\[ \Delta T = P \times R_{TH} \]

• Noise thermometry
  – Bunyan 1992
• Gate electrode resistance thermometry
  – Mautry 1990; Goodson/Su 1994
• Pulsed I-V measurements
  – Jenkins 1995, 2002
• AC conductance measurement
  – Lee 1995; Tenbroek 1996; Reyboz 2004; Jin 2001

Note: these are electrical, non-destructive methods

Noise Thermometry

Bunyan, EDL 13, 279 (1992)

• Body-contacted SOI devices
  – \( L = 0.87 \) and 7.87 \( \mu \)m
  – \( t_{ox} = 19.5 \) nm, \( t_{Si} = 0.2 \) um, \( t_{BOX} = 0.42 \) um
• Bias back-gate \( \rightarrow \) accumulation (\( R \)) at back interface
• Mean square thermal noise voltage \( \langle v_r^2 \rangle = 4k_B T R_B \)
• Frequency range \( B = 1-1000 \) Hz
• Measure \( R \) and \( \langle v_r \rangle \) at each gate & drain bias
• \( T = T_0 + R_{TH} I_D V_D (R_{TH} \sim 16 \) K/mW)
Gate Electrode Resistance Thermometry

Mautry 1990; Su-Goodson 1994-95

- Gate has 4-probe configuration
- Make usual I-V measurement...
- Gate R calibrated vs. chuck T
- Measure gate R with device power P
- Correlate P vs. T and hence $R_{TH}$

Pulsed I-V Measurement

Jenkins 1995, 2002

- Normal device layout
- 7 ns electric pulses, 10 µs period
- Device can cool during long thermal time constant (50-100 ns)
- High-bandwidth (10 GHz) probes
- Obtain both thermal resistance and capacitance
AC Conductance Measurement

Lee 1995; Tenbroek 1996; Reyboz 2004; Jin 2001

- No special test structure
- Measure drain conductance
- Frequency range must span all device thermal time constants
- Obtain both $R_{TH}$ and $C_{TH}$
- Thermal time constant ($R_{TH}C_{TH}$) is bias-independent

$$g_{DS} = \frac{\partial I_D}{\partial V_{DS}}$$

Device Thermometry Results Summary

High thermal resistances:
- SWNT due to small thermal conductance (very small $d \sim 2$ nm)
- Others due to low thermal conductivity, decreasing dimensions, increased role of interfaces

Power input also matters:
- SWNT $\sim 0.01 \text{--} 0.1$ mW
- Others $\sim 0.1 \text{--} 1$ mW