EE 116 Lecture 9

Temperature Dependence of Carrier Concentrations

- https://truenano.com/PSD20/contents/toc2.htm
- Read Ch. 2.6, in particular Ch. 2.6.3 and 2.6.4
- Read Ch. 2.7, up to 2.7.3

- So far: how to get electron & hole concentrations at:
  - Any temperature
  - Any doping level
  - Any energy level
- Previously we also saw:

$$n_i^2 = n_0 p_0 = N_C N_V e^{-E_G/kT} \qquad \text{(mass action law)}$$

- Where

$$N_C = 2\left(\frac{2\pi m_n^* kT}{h^2}\right)^{3/2}$$

- And

$$N_V = 2\left(\frac{2\pi m_p^* kT}{h^2}\right)^{3/2}$$

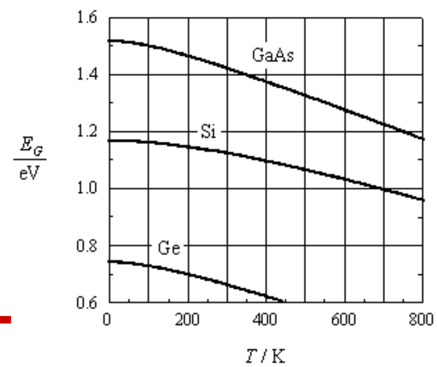
- So the *intrinsic* carrier concentration at *any T* is:

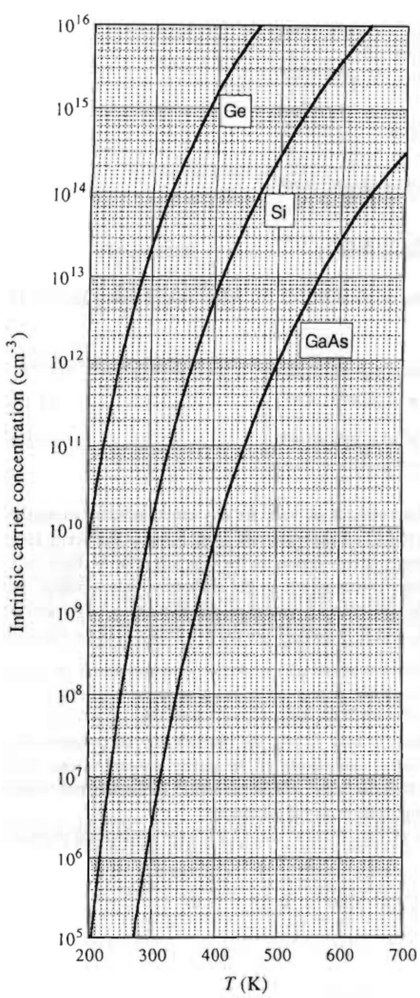
$$n_i(T) = 2\left(\frac{2\pi kT}{h^2}\right)^{3/2}\left(m_n^* m_p^*\right)^{3/4} e^{-E_G/(2kT)}$$

- What does this tell us?
- Note that $m_n^* \approx 1.1 m_0$ and $m_p^* \approx 0.8 m_0$ in silicon
- These are density of states effective masses in Si, not to be confused with conduction effective masses ($F = m^* a$)

- Does the band gap E_G change with T?

→ yes, but we can typically ignore this unless the T range is quite high





Si

T(°C)	n_i(cm^{-3})
0	8.86×10^8
5	1.44×10^9
10	2.30×10^9
15	3.62×10^9
20	5.62×10^9
25	8.60×10^9
30	1.30×10^{10}
35	1.93×10^{10}
40	2.85×10^{10}
45	4.15×10^{10}
50	5.97×10^{10}
300 K	1.00×10^{10}

GaAs

T(°C)	n_i(cm^{-3})
0	1.02×10^5
5	1.89×10^5
10	3.45×10^5
15	6.15×10^5
20	1.08×10^6
25	1.85×10^6
30	3.13×10^6
35	5.20×10^6
40	8.51×10^6
45	1.37×10^7
50	2.18×10^7
300 K	2.25×10^6

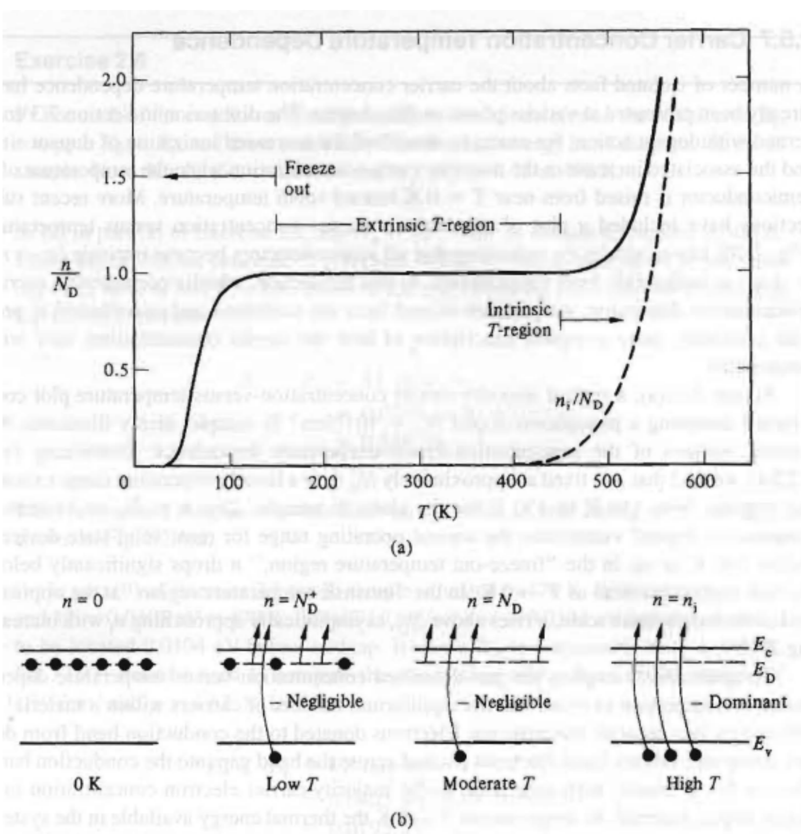
- Plot $\log_{10}$ of n_i vs. T

- What do we expect?

- Note our online "book" plots this vs. 1000/T in Fig. 2.6.4; why?

→ also note the effect of $E_G(T)$ in online plot

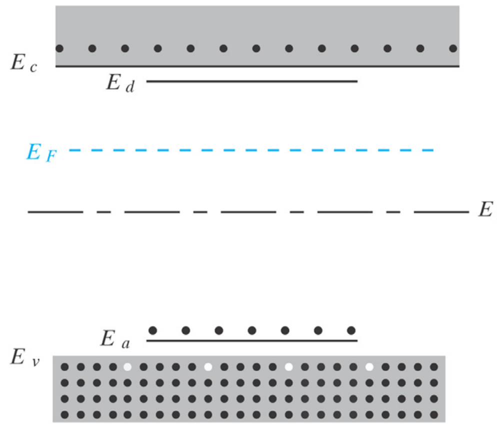
- Recall n_i is very temperature-sensitive! Ex: in Silicon:
  - While T = 300 → 330 K (10% increase)
  - n_i = ~10^{10} → ~10^{11} cm^{-3} (10x increase)
- Also note:
  - Now we can calculate the equilibrium electron (n_0) and hole (p_0) concentrations at any temperature
  - Now we can calculate the Fermi level (E_F) position at any temperature
- Ex: Calculate and show position of Fermi level in doped Ge (10^{16} cm^{-3} n-type) at -15 °C, using previous plot



- Assume Si sample doped with N_D = 10^{15} cm^{-3} (n-type)
- *How does n change with T?* (online book plots this vs. 1000/T in Fig. 2.6.9)
- Recall the band diagram, including the donor level.
- Note three distinct regions:
  - Low, medium, and high-temperature

- So far, we assumed material is either just n- or p-doped and life was simple. At most moderate temperatures:
  - $n_0 \approx N_D$, or
  - $p_0 \approx N_A$, i.e. when the material is extrinsic (useful region)

- What if a piece of Si contains BOTH dopant types? This is called compensation



- Group V elements are donors

and introduce electrons

- Group III elements are acceptors

and introduce holes

- Case I, assume we dope with $N_D > N_A$
  - Additional electrons and holes will recombine until you have $n_0 \approx N_D - N_A$ and $p_0 \approx$ __________

- Case II, what if we introduce $N_D = N_A$ dopants?
  - The material once again becomes _____________ and $n_0 \approx p_0 \approx$ ______

- Case III and more generally, we must have charge neutrality in the material, i.e. positive charge = negative charge, so $p_0 + N_D =$ _____________

- So most generally, what are the carrier concentrations in thermal equilibrium, if we have both donor and acceptor doping?

$n_0 p_0 = n_i^2$ (mass-action law)

$n_0 + N_A = p_0 + N_D$ (charge neutrality)

two equations with two unknowns → we can solve for n_0 and p_0 as $f(N_A, N_D, n_i)$

- And how do these simplify if we have $N_D >> N_A$ (n-type doping dominates)?
- When is ">>" approximation OK?

EE 116 Lectures 10-11

Carrier drift, Mobility, Resistance

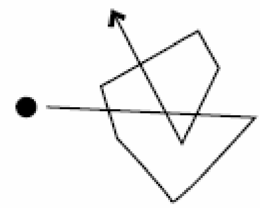
Also see CCH Ch. 2

- Let's recap 5-6 major concepts so far:

- Memorize a few things, but recognize many.
  - Why? Semiconductors require lots of approximations!

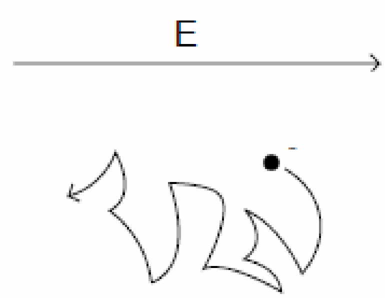
- Why all the fuss about the abstract concept of E_F?
  - Consider (for example) *joining* an n-doped piece of Si with a p-doped piece of Ge. How does the band diagram look?

- So far, we've learned effects of temperature and doping on carrier concentrations
- But no electric field = not useful = boring materials
- The secret life of C-band electrons (or V-band holes): they are essentially free to move around, how?



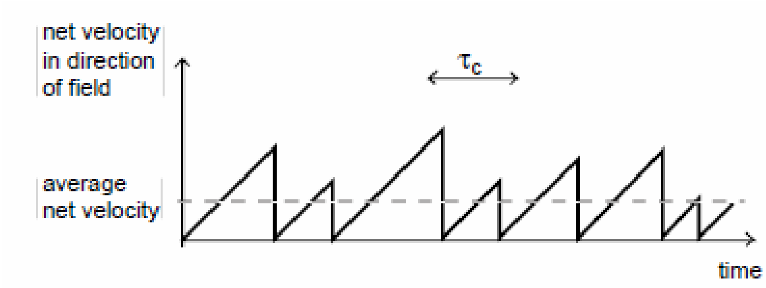
- Instantaneous velocity given by thermal energy:
- Collision time (with what?) is of the order $\tau_C \sim 0.1$ ps
- So average distance (mean free path) travelled between scattering events $L \sim v_T \tau_C$

- But with no electric field (E=0) total distance travelled is: ___
- So turn ON an electric field:



- $F = \pm qE$
- $F = m^* a \rightarrow a = \pm qE/m_{n,p}^*$
- Between collisions, carriers accelerate along E field:
  - $v_n(t) = a_n t = -qEt/m_n$ for electrons
  - $v_p(t) = a_p t = +qEt/m_p$ for holes
- Recall how to draw this in the energy band picture

- On average, velocity is randomized again every τ_C (collision time)



- So average velocity in *E*-field is: $|v| = \frac{qE\tau_C}{m^*}$

- We call the proportionality constant the carrier <u>mobility</u>

$$\mu_{n,p} = \frac{q\tau_C}{m^*_{n,p}} \qquad |v_{n,p}| = \mu_{n,p}E$$

- This is a very important result!!! (what are the units?)
- What are the roles of $m_{n,p}$ and τ_C?

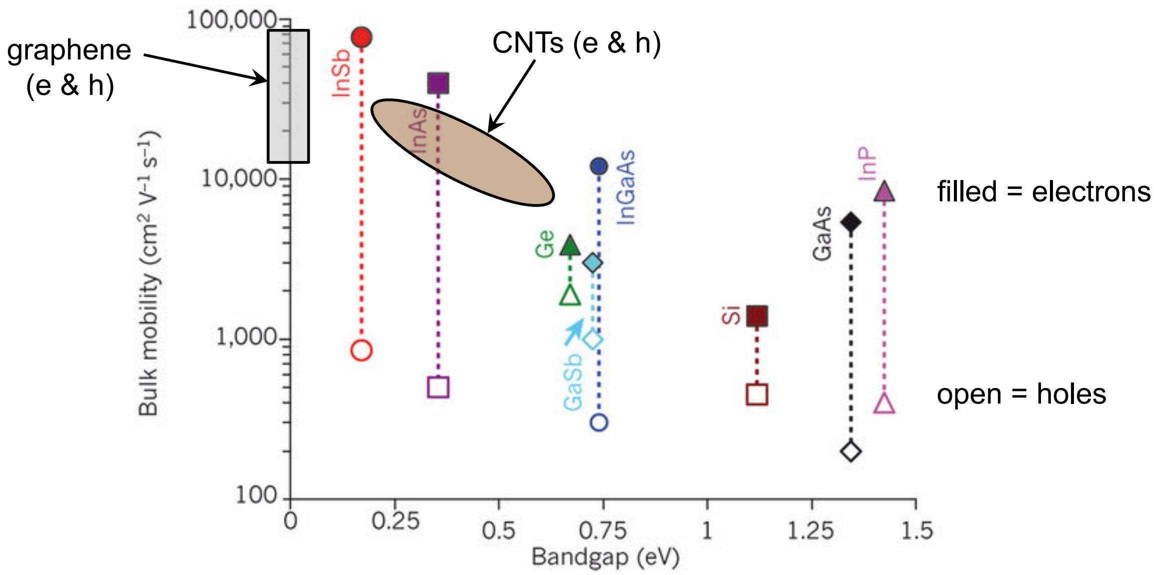
- Then for electrons: $v_n = -\mu_n E$
- And for holes: $v_p = \mu_p E$

- Mobility is a measure of *ease of carrier drift in E-field*
  - If m↓ "lighter" particle means μ…
  - If τ_C↑ means longer time between collisions, so μ…

- Mobilities of some *undoped* (intrinsic) semiconductors *at room temperature*:

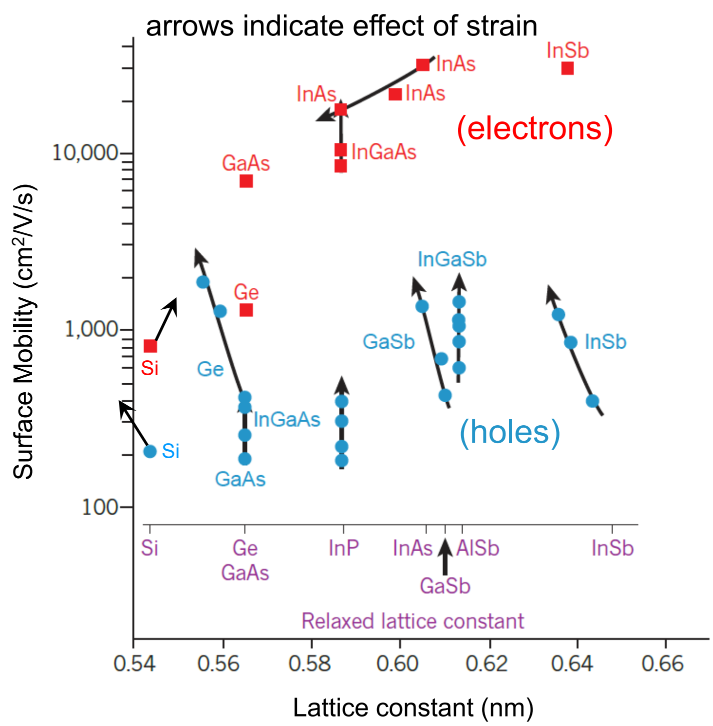
	Si	Ge	GaAs	InAs	Graphene / CNTs
μ_n (cm^2/V·s)	1400	3900	8500	30000	40000
μ_p (cm^2/V·s)	470	1900	400	500	40000

- Mobility of undoped materials vs. band gap
- Why does band gap matter?
- Ge and CNTs have highest μ for holes

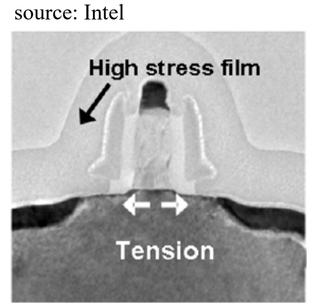


source: R. Pillarisetty, *Nature* (2011)

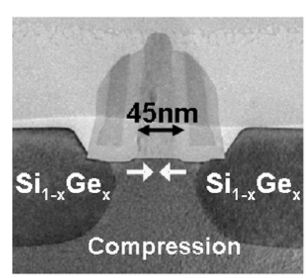
- Surface mobilities are lower (why?)
- Material strain also matters (why?)



source: Intel



(Si nMOS)



(Si pMOS)

source: J. del Alamo, *Nature* (2011)

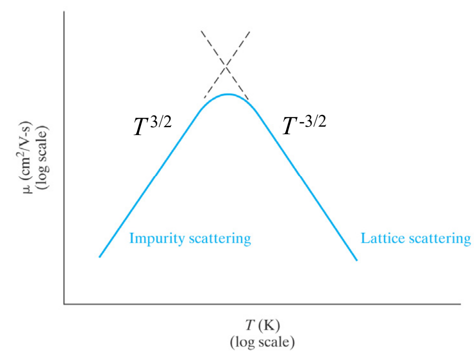
- What does mobility (through τ_C) depend on?
  - Lattice scattering (host lattice, e.g. Si or Ge vibrations)
  - Ionized impurity (dopant atom) scattering
  - Electron-electron or electron-hole scattering
  - Interface (surface) scattering

- Which ones depend on temperature?
- Qualitative, how?

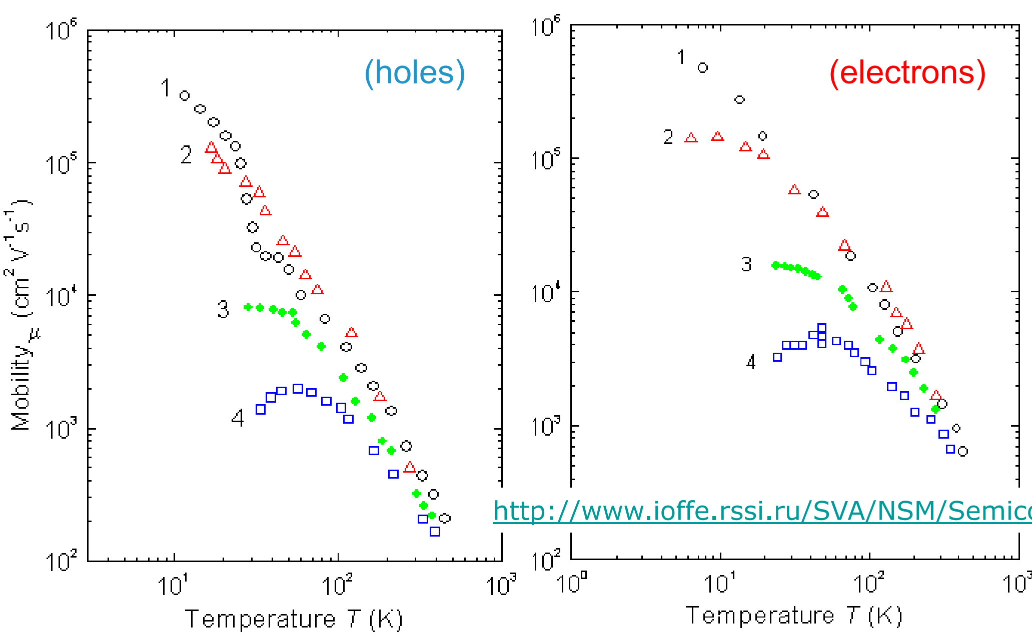
- Strongest scattering, i.e. lowest mobility dominates.

$$\frac{1}{\mu} = \frac{1}{\mu_L} + \frac{1}{\mu_{II}} + \frac{1}{\mu_{e-e}} + \frac{1}{\mu_S} + \cdots$$

- Qualitatively →



- Quantitatively we rely on experimental measurements (calculations are difficult and not usually accurate):



http://www.ioffe.rssi.ru/SVA/NSM/Semicond/Si/electric.html

- Once again, qualitatively we expect the mobility to decrease with total impurities (N_D+N_A)

- Why *total* impurities and not just N_D or N_A? (for electrons and holes?)

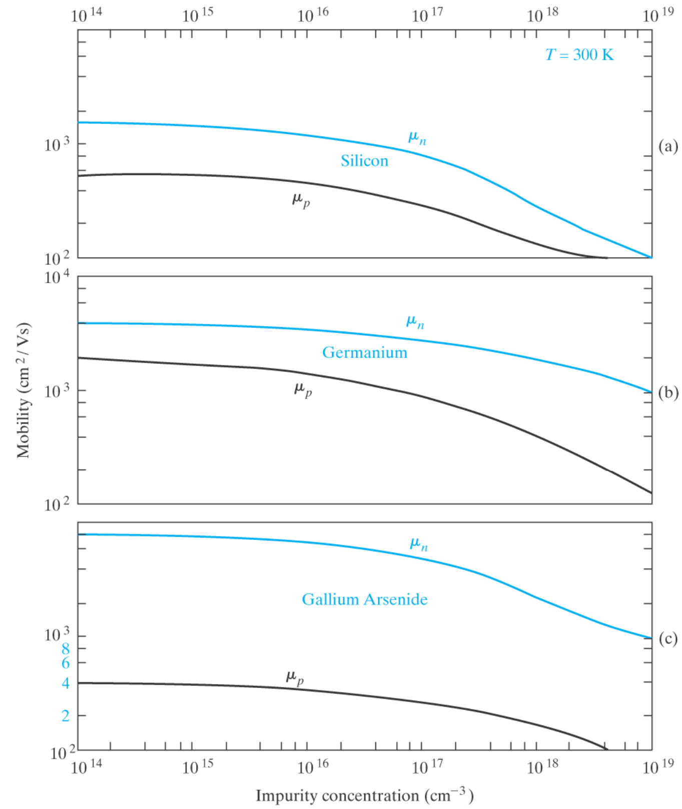
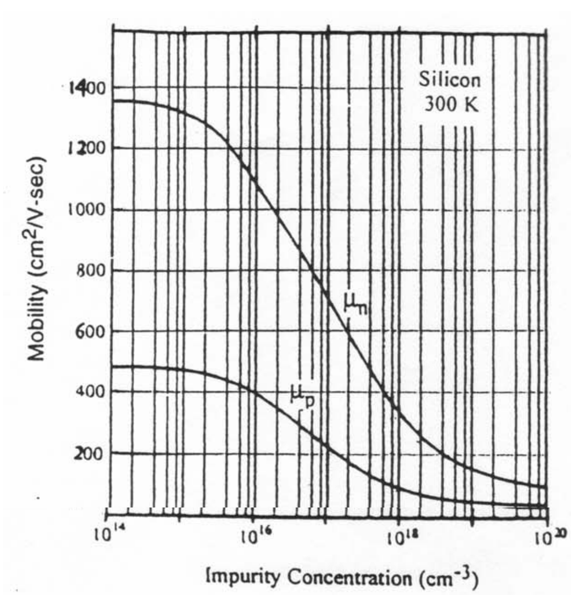


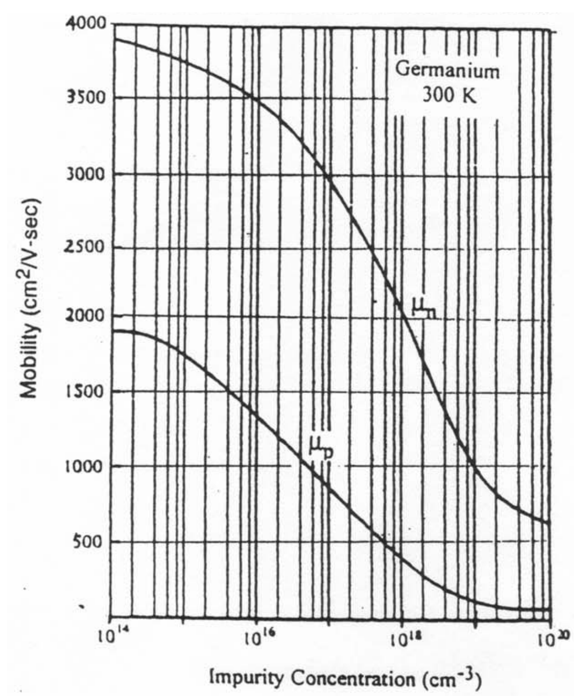
Figure 3.23

Variation of mobility with total doping impurity concentration ($N_a + N_d$) for Ge, Si, and GaAs at 300 K.

- Measured mobility for Si and Ge, in linear scale:



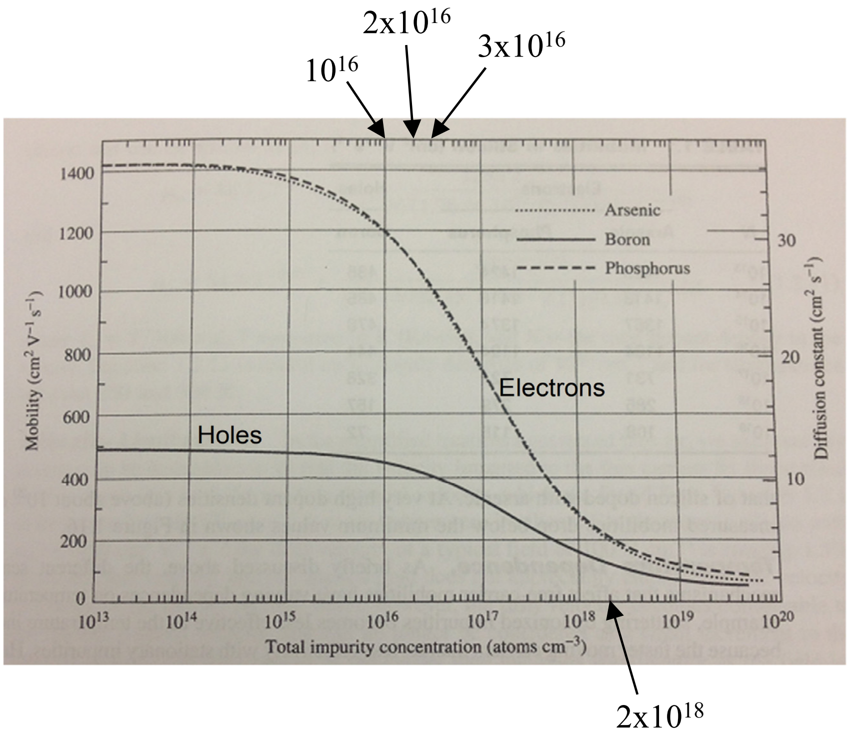
($N_D + N_A$)



($N_D + N_A$)

- Be careful when reading log scale!

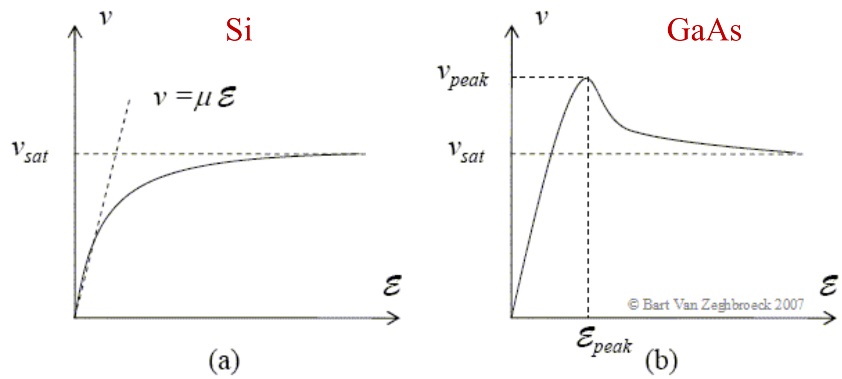
(this diagram is often given to you on midterm & final)



- Ex: What is the hole drift velocity at room temperature in undoped silicon, in a field E = 1000 V/cm ($m_p^* = 0.39m_0$)? What is the average time & distance between collisions?

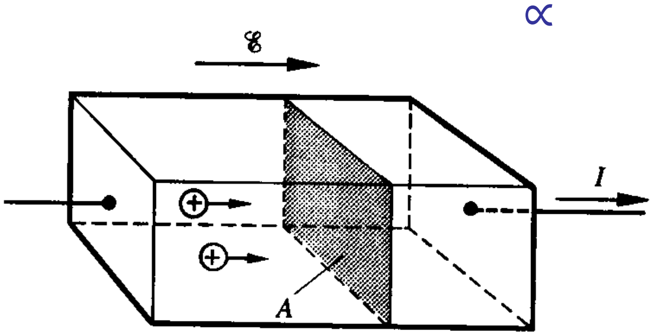
Note: compare v_p (hole drift velocity) with v_T (thermal velocity)

- What about at very high electric fields?
  - $\mathcal{E} > 10^4$ V/cm = 1 V/μm



- The drift velocity saturates due to intense collisions with the lattice. This is equivalent to "terminal velocity" due to air friction for falling objects (e.g. skydivers)
- In Si, we can write empirically: $v(\mathcal{E}) = \frac{\mu\mathcal{E}}{1+\frac{\mu\mathcal{E}}{v_{sat}}}$
- In Si, $v_{sat} \sim 10^7$ cm/s

- Now we can calculate current flow in realistic devices!
- Net velocity of charge particles → electric current
- Drift current density ∝ net carrier drift velocity
  ∝ carrier concentration
  ∝ carrier charge



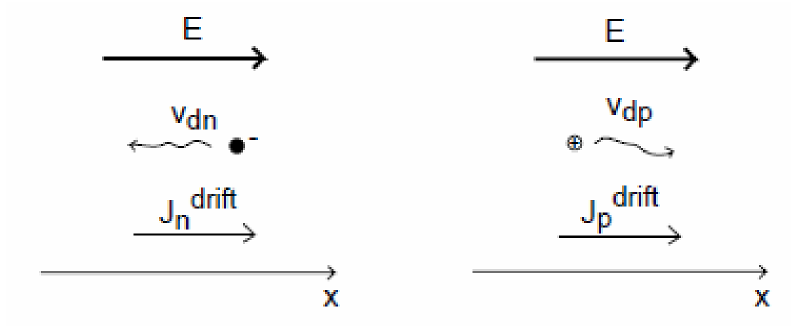
(charge crossing plane of area A in time t)

$$J_n^{drift} = -qnv_{dn} = qn\mu_n E$$

$$J_p^{drift} = +qpv_{dp} = qp\mu_p E$$

- First "=" sign always applies. Second "=" applies typically at low-fields (<10^4 V/cm in Si)

- Check units and signs:



- Total drift current:

$$J^{drift} = J_n^{drift} + J_p^{drift} = q(n\mu_n + p\mu_p)E$$

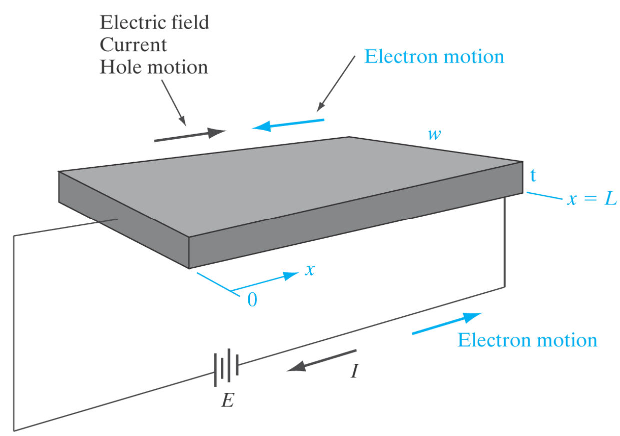
- Has the form of Ohm's Law!
- Current density: $J = \sigma E = \dfrac{E}{\rho}$
- Current: $I = JA = \dfrac{E}{\rho}A = \dfrac{V}{L}\dfrac{A}{\rho} = \dfrac{V}{R}$
- This is very neat. We derived Ohm's Law from basic considerations (electrons, holes) in a semiconductor.

- Resistivity of a semiconductor:

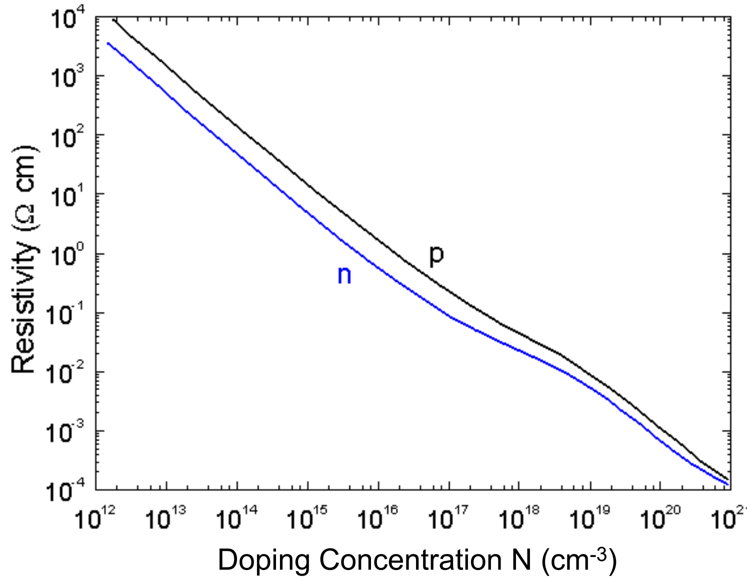
$$\rho = \frac{1}{\sigma} = \frac{1}{q(n\mu_n + p\mu_p)}$$

- What about when n >> p? (n-type doped sample)
- What about when n << p? (p-type doped sample)
- Drift and resistance:

$$R = \rho\frac{L}{wt} = \frac{1}{\sigma}\frac{L}{wt}$$



- Experimentally, for Si at room T:



- This is absolutely essential to show our control over resistivity via doping!

- Notes:
  - This plot does not apply to compensated material (with comparable amounts of <u>both</u> n- and p-type doping)
  - This plot applies most accurately at low-field ($<10^4$ V/cm)