Microstructural origin of resistance–strain hysteresis in carbon nanotube thin film conductors

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A basic need in stretchable electronics for wearable and biomedical technologies is conductors that maintain adequate conductivity under large deformation. This challenge can be met by a network of one-dimensional (1D) conductors, such as carbon nanotubes (CNTs) or silver nanowires, as a thin film on top of a stretchable substrate. The electrical resistance of CNT thin films exhibits a hysteretic dependence on strain under cyclic loading, although the microstructural origin of this strain dependence remains unclear. Through numerical simulations, analytic models, and experiments, we show that the hysteretic resistance evolution is governed by a microstructural parameter $\xi$ (the ratio of the mean projected CNT length over the film length) by showing that $\xi$ is hysteretic with strain and that the resistance is proportional to $\xi^2$. The findings are generally applicable to any stretchable thin film conductors consisting of 1D conductors with much lower resistance than the contact resistance in the high-density regime.

Stretchable conductor | carbon nanotube | resistance-strain hysteresis | coarse-grained molecular statics | cyclic loading

Significance

An essential building block for stretchable electronics, the enabler of novel wearable and biological technologies, is stretchable conductors that can maintain good electrical conductivity under large deformation. A widely used approach to meet this need is to use a network of 1D nanomaterials, such as carbon nanotubes, as a thin film on a stretchable substrate. When these networks are subjected to complex loading paths during fabrication and usage, they must be designed to maintain good electrical conductivity under large deformation. This can be achieved by combining computer simulations, analytic modeling, and experiments, finding that the hysteretic resistance–strain relationship is controlled by a single microstructural parameter $\xi$, the ratio of the mean projected carbon nanotube length over the film length.


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a Lennard-Jones potential (17, 31–33) (Methods and SI Appendix, section S1). The long-range attraction and short-range repulsion of the substrate are modeled by an external potential applied to all nodes (SI Appendix, section S1). Periodic boundary conditions are applied in the $x$ and $z$ directions (i.e., within the plane of the CNT film) (Fig. 1A). For simplicity, the initial configurations of the CNTs were created as randomly oriented straight lines parallel to the substrate surface. Stretching of the CNT film is simulated by elongating the simulation cell in the $x$ direction and contracting in the $z$ direction in small increments, allowing all nodes to relax to a local energy minimum after each increment based on the conjugate-gradient algorithm. Fig. 1C shows an example of the morphology change of the CNT film during cyclic loading from the CGMS simulations. The initial simulation cell has in-plane dimensions of $H_x = H_z = 1,200$ nm and contains $N_{\text{CNT}} = 135$ CNTs, each with length $L_{\text{CNT}} = 2,400$ nm and diameter $d_{\text{CNT}} = 1$ nm. It is interesting to note that, after a few strain cycles, the predicted CNT network structure becomes progressively more similar to the SEM observations (Fig. 1B). In particular, the CNTs become more curved and form thicker bundles. We observe that, during loading, some CNT bundles that are well-aligned to the $x$ direction break apart through an unzipping process, allowing the CNTs to move away from each other (SI Appendix, Fig. S2)—for brevity, we shall refer to this mechanism as sliding between CNTs. During unloading, these CNTs buckle (Movie S1). When the CNT film is stretched again in the $x$ direction, the CNTs first straighten out (i.e., undoing the buckling
The evolution of the relative sheet resistance is shown in SI Appendix, Fig. S7.

The predicted resistance–strain curves (Fig. 2 C and D and SI Appendix, Fig. S6) show striking resemblance with the experimental data (Fig. 2 A and B) in both x and z directions. The remaining differences between the CGMS predictions and experimental results (e.g., in the transverse direction) can be attributed to the idealizations in the simulation model, including straight CNTs as initial configurations, uniform length of CNTs, and the simplified description of the CNT–substrate interaction.

The close agreement between the predictions and measurements, especially in the longitudinal direction, suggests that the microstructural feature controlling the resistance–strain evolution can be determined by analyzing the simulation results. To pinpoint the controlling microstructural feature, we examined a large number of candidate features of the predicted CNT network, such as mean orientation, mean bending (SI Appendix, Fig. S8), number of contacts per CNT (SI Appendix, Fig. S9), etc. The parameters that show the strongest correlation with the resistances are the mean relative lengths of CNTs projected in the x direction \( \xi = (l_x) / l_t \) and in the z direction \( \xi_z = (l_z) / l_t \), respectively (SI Appendix, Fig. S6 C and D), where \( l_t \) means average over all CNTs. For convenience, it is also useful to consider their inverse: \( \eta_x = 1 / \xi \) and \( \eta_z = 1 / \xi_z \). Fig. 4 A and B show that the relative changes of \( \eta_x \) and \( \eta_z \) with strain exhibit hysteresis and closely resemble the resistance–strain curves. Both \( \eta_x \) and \( \eta_z \) exhibit strong correlation with the resistances \( R_x \) and \( R_z \) respectively (SI Appendix, Fig. S11). This strongly suggests that \( \xi_x \) and \( \xi_z \) are the controlling microstructural parameters for which we are looking.

To prove the hypothesis that \( \xi_x \) and \( \xi_z \) (or equivalently, \( \eta_x \) and \( \eta_z \)) are indeed the microstructural features responsible for the hysteretic resistance–strain behavior, we need to explain (i) why they exhibit hysteresis in cyclic loading and (ii) how they control electrical resistance. To answer the first question, we note that, during the first loading phase, \( l_t \) increases with the strain due to CNT reorientation (SI Appendix, Fig. S10 A and C). However, the amount of increase is not as large as that of the film size \( h_s \), so that \( \eta_x \) increases with strain (Fig. 3 A). During unloading, the CNTs buckle (Fig. 4 A and B), and \( l_t \) decreases in proportion to that of the film size \( h_s \), so that \( \eta_x \) stays nearly constant. The
initial loading as well as the subsequent loadings beyond the previous maximal stretch, the maximal stretch equals the current stretch (i.e., $\lambda_m = \lambda$) (Fig. 4C). However, during unloading, $\lambda$ decreases, while $\lambda_m$ stays at the maximal stretch value, so that $\lambda_m > \lambda$ (Fig. 4C); $\theta_m$ is the new orientation angle under $\lambda_m$; $\theta_m = \arctan(\tan(\Theta))^{-1/2}$ (SI Appendix, section S3). Fig. 3 C and D shows the evolution of $\Delta R_0/\eta_0$ and $\eta R_0/\eta_0$ during three loading and unloading cycles predicted by these analytic expressions. The analytic results are in excellent agreement with the data extracted from the CGMS simulations, which are shown in Fig. 3 A and B (SI Appendix, Fig. S12).

We now address the second question of how $\eta_1$ and $\eta_2$ control the resistances $R_1$ and $R_2$. We note that, in this case, where the contact resistance $R_{contact}$ is much higher than the intrinsic CNT resistance $R_{CNT}$, the CGMS model shows that resistance of the film is very well-described by the following expressions:

\[ R_1 = a R_{contact} \eta_1^2/N_{CNT}, \]  
\[ R_2 = a R_{contact} \eta_2^2/N_{CNT}, \]

where $a$ is a dimensionless variable related to the morphology of the CNT network (SI Appendix, section S3). The quadratic dependence of resistance $R$ on parameter $\eta$ can be qualitatively understood using the following model. Since the overall resistance is dominated by contact resistance, for simplicity, we assume that the electrical resistance along each CNT is zero. In other words, a charge can travel over an average distance of $l_0/\eta$ in the $x$ direction without experiencing any resistance. In this limit, the dimensionless parameter $\xi_1 = (l_0/\eta)/h_0$ is analogous to the Knudsen number in fluid mechanics. For a charge to travel from one end to the other end of the film in the $x$ direction, the least resistance path that it can take should contain $\xi_1 = h_0/\lambda$ CNTs on average. The resistance of this path is $R_{contact}/N_{CNT}$. Given a total number of $N_{CNT}$ nanotubes, we can consider the entire CNT network consisting of $N_{CNT}$ parallel paths, each containing $\eta_1$ nanotubes. As a result, the overall resistance of the CNT film is $R_{contact}/N_{CNT}/\eta_1 = R_{contact}/N_{CNT}/\eta_1$. This is very similar to Eq. 3, in which a dimensionless parameter $a$ is introduced to account for the error induced by replacing the CNT network with a collection of parallel paths (SI Appendix, section S3).

The CGMS simulation results of the resistance $R$ in both the stretching and transverse directions show linear correlation with $\eta^2/N_{CNT}$, with the fitting coefficient $a = 0.125$ (Fig. 5 A and B). Combining Eqs. 1-4, we arrive at an analytic model for the evolution of the resistance in both the stretching and transverse directions: $R_1$ and $R_2$. The relative change of the resistance $\Delta R_1/R_1$ and $\Delta R_2/R_2$ (Fig. 5 C and D) predicted by the analytic model shows good agreement with the CGMS simulation results in Fig. 2 C and D.

Based on the good agreement between the experimental data, CGMS model, and analytic theory, we conclude that $\eta_1$ and $\eta_2$ are indeed the controlling microstructural parameters for the hysteretic resistance–strain behavior. Intuitively, we might expect the number of contacts between CNTs to be an important microstructural parameter for resistance. However, the CNT films in this study are well above the percolation limit, so that the contacts between CNTs are redundant. This is why considering the CNT network as parallel and isolated paths (and significantly reducing the number of contact points in this process) still captures the relative resistance change of the film very well. Indeed, our simulation results do not show a good correlation between the number of contacts and resistance (SI Appendix, Fig. S9). Varying the density of CNTs also has a negligible effect on the relative resistance change–strain curves (SI Appendix, Fig. S13).

Using the CGMS model, we can also predict the effect of CNT length on the resistance change of the network under cyclic loading. Fig. 6 shows the resistance change–strain relation in the stretching direction for CNTs with lengths (i) 800, (ii) 1,600, and (iii) 2,400 nm. Here, we assume that the CNT resistance depends on parameter $\eta$ as the asymmetric behavior of the CNTs between reorientation and sliding during loading and buckling during unloading. (A) Schematic of the asymmetric behavior of the CNTs between reorientation and sliding during loading and buckling during unloading. This leads to the hysteretic microstructural parameter $\xi$ and therefore, resistance of CNTs between loading and unloading. (B) CGMS simulation results showing CNTs reorienting and sliding during loading and buckling during unloading in the same region. (C) In the analytic model of the evolution of $\xi$ (Eqs. 1 and 2), during loading and subsequent loading beyond the maximal stretch reached before, the maximal stretch $\lambda_m$ equals $\lambda$ while during unloading, $\lambda_m$ is the maximal stretch ever reached and $\lambda < \lambda_m$. hysteresis of $\eta_1$ is due to the asymmetric behavior of CNTs between reorientation and sliding during loading and buckling during unloading (Fig. 4 A and B). A similar trend is observed in the $z$ direction, although the amplitude of $\Delta \eta_1/\eta_0$ is much smaller than that of $\Delta \eta_2/\eta_0$ (SI Appendix, Fig. S10 B and D).

To be more quantitative, we construct an analytic model for the evolution of the $\eta$ parameters with strain. For simplicity, we consider a collection of CNTs in which the end-to-end vector of each CNT has an orientation angle $\Theta$ (relative to the $x$ axis) that is randomly distributed. We first consider how the end-to-end vectors of all CNTs would vary if they deform affinely as the film is stretched along $x$ by a stretching ratio $\lambda$ and compressed along $z$ by a factor of $1/\sqrt{\lambda}$. In this case, the end-to-end vectors with an initial orientation angle above a critical angle $\Theta_1 = \arccos(1/(\lambda^2 + 1))$ would become shorter, and the vectors with initial orientation angle below the critical angle would become longer. We assume that CNTs can easily accommodate a reduction of end-to-end distance by buckling, so that CNTs oriented above the critical angle would indeed deform affinely. However, CNTs oriented below the critical angle would not deform affinely, because doing so would require their end-to-end distance to become longer. Instead, we assume that the end-to-end vectors for these CNTs will only rotate to the new orientation $\Theta = \Theta_m \arctan(\tan(\Theta))^{-1/2}$ but that their lengths will remain unchanged. This will cause sliding between CNTs previously in the same bundle. During unloading, after reaching the maximal stretch $\lambda_m$, the CNTs with $\Theta > \Theta_1$ reversibly recover from the buckling, while the CNTs with $\Theta < \Theta_1$ buckle. Based on these assumptions, we obtain analytic expressions of $\xi_1$ and $\xi_2$ as a function of strain:

\[ \xi_1 = \frac{2}{\pi h_0} \left( \int_0^{\Theta_1} \cos \Theta_m \frac{\lambda}{\lambda_m} d\Theta + \int_{\Theta_1}^{\pi/2} \cos \Theta d\Theta \right), \]
\[ \xi_2 = \frac{2}{\pi h_0} \left( \int_0^{\Theta_1} \sin \Theta_m \frac{\sqrt{\lambda^2 \cos^2 \Theta + \sin^2 \Theta/\lambda}}{\lambda_m} d\Theta + \int_{\Theta_1}^{\pi/2} \sin \Theta d\Theta \right), \]

where $l_{CNT} = \sqrt{\lambda^2 \cos^2 \Theta + \sin^2 \Theta/\lambda}$ is the length of the CNTs under affine deformation following the substrate. During the
Effect of the CNT length on the resistance change: During loading is caused by the combination of buckle causes $\xi$ to stay nearly constant during unloading. In the limit of high contact resistance, the electrical resistance of the film is proportional to $\xi^{-2}$, where $\xi$ acts as a “mean free path” relative to the film dimension along which the charge can travel without experiencing Ohmic loss at the CNT contacts. Our simulations further predict that CNTs with smaller lengths or larger diameters exhibit smaller hysteresis of resistance change on a loading and unloading cycle. However, the CNT density itself has a minimal effect on the relative resistance change with strain when the CNT network is far above the percolation threshold. We believe that these conclusions are not limited to CNT films but are generally applicable to stretchable conductors consisting of a network of 1D tubes or wires with individual resistance that is much lower than the contact resistance. Furthermore, our numerical (CGMS) model has even broader applicability and is not limited by the relative magnitude of the two resistances. Our predictions on the microstructural origin of resistance hysteresis can potentially be validated more directly if several existing experimental challenges can be overcome, such as the identification of individual CNTs in an SEM image to measure the end-to-end distances of CNTs and the fabrication of long CNTs with more precisely controlled lengths and diameters.

Methods

Spray Coating CNT Films. We created a conducting thin film of CNTs by spray coating a well-dispersed CNT solution onto a PDMS substrate. The PDMS (Dow Corning Sylgard 184) with 15:1 base to cross-linker ratio was mixed, degassed, and cured overnight at 80 °C. The PDMS substrate had thickness around 1 mm and was cut into rectangles with dimensions 7.62 × 1.27 cm.

To prepare the CNT solution, we used single-wall P2 CNTs (Carbon Solution, Inc.) of diameters 1.2–1.7 nm were ultrasonicated in NMP (Fisher Scientific) with a Cole Parmer 750-W tip sonicator at 30% power for 30 min. The solution was then centrifuged (Servall Lynx 4000, Fibelite F21-850V Rotor, Thermo Fisher Scientific) for 30 min at 8,000 rpm to remove large bundles and amorphous carbon. The top 75% of the solution was used for spray coating.

The CNT solution was spray coated with a commercial airbrush (Master Airbrush; model SBB44-SET). Before the spray coating, the PDMS substrates were activated with UV ozone for 20 min. During the spray coating, the hot plate underneath the PDMS was held at 200 °C so that the solvent (boiling point ~180 °C) evaporates during deposition. We patterned the CNT film into a small square with dimension 0.51 × 0.51 cm in the center of the substrate by using a mask cut with the Silhouette Cameo 3 machine (Silhouette America, Inc.). Multiple passes of airbrush (~50–100 times) were performed to spray CNTs until the two-point resistance reached around 5 kΩ.

Measuring Resistance of CNT Films. Two 30-nm-thick gold stripes were patterned and evaporated onto the two opposite edges of the CNT film to ensure a good electrical contact (43). We tested the resistance measurement of CNT films with different lengths between the two gold stripes and found that the resistance almost linearly scaled with the length; therefore, the contact resistance was negligible. Then, the patterned gold patches were connected to an Agilent E4980A LCR meter (Agilent Technologies, Inc.) via liquid metal EGain forming a stretchable conductive path (SI Appendix, Fig. S1). We applied loading and unloading cycles to the PDMS substrate with a home-built
mechanical strain platform and in situ measured the resistance in the stretching and transverse directions using the LCR meter. In a typical experiment, we stretched the sample along the length direction to 20% strain, fully unloaded it, and then repeated this process for another two cycles, with the maximal strains being 40 and 60%, respectively.

Observing CNT Films Under SEM. In situ SEM strain measurements were conducted in the FEI Magellan Scanning Electron Microscope at 1 kV with a 13-pA beam current. The sample was clamped down with clips and measured before, during, and after stretching the sample. Due to the insulating nature of the flexible substrates, scan times were adjusted to prevent deformation of the CNT network caused by localized heating of the PDMS substrate from the electron beam.

CGMS Simulation of CNT Films. We used the CGMS method to simulate the morphology change of CNTs under cyclic loading with increasing levels of strain with a molecular dynamics package MD++. A thin sheet of single-wall CNTs was simulated by a collection of CNTs interacting with each other and with the substrate. Each CNT was represented by a series of nodes (on the order of 100) connected by linear segments. The interactions between nodes were introduced to reproduce the stretching and bending stiffness of the CNTs and van der Waals interactions between CNTs (SI Appendix, section S1). The interactions between the CNTs and the substrate were modeled as an external potential with a long-range attraction and a short-range repulsion applied to all nodes (SI Appendix, section S1).

For simplicity, the initial configurations of the CNTs were created as randomly oriented straight lines parallel to the substrate surface. This structure was then relaxed to a local energy minimum using the conjugate-gradient algorithm, so that the CNTs form a thin sheet on the substrate. The simulation cell sizes in the x and z directions were allowed to adjust during the relaxation, so that a zero-stress state was reached. The initial simulation cell had the sizes of $H_x$, $H_y$, and $H_z$ in the x, y, and z directions, respectively. Stretching was simulated by elongating the simulation cell size in the x direction to $H_x = H_x (1 + \varepsilon)$ in small increments (0.1% strain for each increment), allowing nanotubes to relax to a local energy minimum after each increment. The simulation cell size in the z direction was adjusted in each strain increment to $H_z = H_z (1 + \varepsilon)^{1/2}$ to ensure the uniaxial loading condition of the incompressible PDMS substrate (SI Appendix, section S1).

The simulation cell is subjected to the same loading and unloaded cycles as in the experiments. The simulation example that we show in Fig. 1C has the initial size of simulation cell as $H_x = H_z = 1,200$ nm and $H_y = 100$ nm. The convergence of the simulation with respect to the simulation cell size is shown in SI Appendix, Fig. S3.

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Supporting Information for
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S1. Pair potential for CGMS simulation

To understand the resistance-strain hysteresis of carbon nanotubes (CNTs) conductors, we use coarse-grained molecular statics (CGMS) method to simulate the morphological change of CNT networks under loading cycles. We model a thin sheet of CNTs composed of a collection of $N_{\text{CNT}}$ nanotubes, with each one discretized by a set of (on the order of 100) $N_{\text{node}}$ nodes. We use the following pair potential for different nodes $(1, 2)$:

$$V(r_i, r_j) = \sum_i \frac{1}{2} k_s \left( |r_{i+1} - r_i| - l_0 \right)^2 + \sum_i \frac{1}{2} k_B \left( \frac{(r_{i+1} - r_i) \cdot (r_{i+1} - r_j)}{|r_{i+1} - r_i| |r_{i+1} - r_j|} + 1 \right) + \sum_{i \neq j} \frac{C_{12}}{|r_i - r_j|^{12}} - \frac{C_6}{|r_i - r_j|^{6}}.$$

(S1)

The first two terms are the stretching and bending energies between neighboring nodes in one CNT. The interaction parameters are chosen to reproduce the bending and stretching response of an elastic tube with Young’s modulus $E_{\text{CNT}} = 5\text{TPa}$, inner diameter $d_{in} = d_{\text{CNT}} - h_{\text{CNT}}$, and outer diameter $d_{out} = d_{\text{CNT}} + h_{\text{CNT}}$, where $d_{\text{CNT}}$ is the diameter of the CNTs, and we adopt the CNT wall thickness $h_{\text{CNT}} = 0.07 \text{ nm}$ (3). The parameters $r_i$ in Eq. S1 is the position of a node, $l_0 = l_{\text{CNT}} / N_{\text{node}}$ is the stress-free length between two neighboring nodes, with $l_{\text{CNT}}$ the length of CNTs. The stretching stiffness is $k_s = E_{\text{CNT}} A_{\text{CNT}} / l_0$ with the cross-section area $A_{\text{CNT}} = \pi \left( r_{\text{out}}^2 - r_{\text{in}}^2 \right) / 2$, and the bending stiffness is $k_s = E_{\text{CNT}} I_{\text{CNT}} / l_0$ with the moment of inertia $I_{\text{CNT}} = \pi \left( d_{\text{out}}^4 - d_{\text{in}}^4 \right) / 64$. The last term of Eq. S1 is the Lennard-Jones potential for non-neighboring nodes. The
parameters $C_{12} = N_c c_{12}$ and $C_o = N_c c_o$, where $N_c$ is the number of carbon atoms represented by each node (4), $c_{12} = 2516582.4$ kcal · mol$^{-1}$ Å$^{12}$, and $c_o = 1228.8$ kcal · mol$^{-1}$ Å$^{6}$ (5). For single-wall CNTs, we estimate $N_c = \pi d_{CNT} l_0 / A_c$, where $A_c = 2.6194$ Å$^2$ is the average area covered by each carbon atom. An additional potential is applied to each node to model the adhesion and repulsion of the substrate

\[
U = \begin{cases} 
\frac{1}{2} k_1 (y - y_0)^2 & y \leq y_0, \\
 k_2 \left(1 - e^{-\frac{1}{2}(y-y_0)^2}\right) & y > y_0, 
\end{cases}
\]  

(S2)

where $y$ is the thickness direction of the CNT network, and $y_0$ is the position of the substrate. Here we set $k_1 = k_2 = k_y$. The initial simulation cell has the size of $H_x$, $H_y = 100$ nm and $H_z$ in the $x$, $y$ and $z$ direction. Periodic boundary conditions are applied in the $x$ and $z$ directions, i.e. within the plane of the CNT film.

**S2. Calculating the resistance of CNT networks**

**S2.1. Zero net current condition**

According to Ohm’s law, the electrical current $I$ is proportional to the voltage difference across a conductor $\Delta V$

\[
I = G \Delta V ,
\]  

(S3)

where $G$ is the conductance. Fig. Sec S1 sketches the electrical current through one CNT, which is discretized with multiple nodes. A voltage drop is applied at the two ends of the simulation cell. When the steady state is reached, net current going in and out each node is zero. Take node 3 as an example, and this condition can be expressed as

\[
G_{32} (V_3 - V_2) + G_{34} (V_3 - V_4) = 0. 
\]  

(S4)

This can be reorganized as

\[
(G_{32} + G_{34}) V_3 - G_{32} V_2 - G_{34} V_4 = 0 . 
\]  

(S5)

The general condition of zero net current for node $i$ is

\[
\sum_j G_{ij} V_i - \sum_j G_{ij} V_j = 0 , 
\]  

(S6)

where nodes $j$ are the ones connecting node $i$. 
Fig. Sec S1. Schematics of the current through the nodes of a CNT in the simulation cell and the two neighboring unit cells under the periodic boundary condition.

S2.2. Periodic boundary condition

Since the periodic boundary condition is applied to the simulation cell and a voltage difference $\Delta V_0$ is applied at the two ends of the simulation cell, the voltage of node 1 in the simulation cell and node 1' which physically connected to node 2 (Fig. Sec S1) satisfies the relation

$$ V'_1 = V_1 + \Delta V_0. \quad (S7) $$

The zero net current condition for node 2 is

$$ G_{21} (V_2 - V'_1) + G_{23} (V_2 - V_3) = 0, \quad (S8) $$

which can be rewritten as the same form of Eq. S6, but with a source term $G_{21} \Delta V_0$

$$ G_{21} (V_2 - V'_1) + G_{23} (V_2 - V_3) = G_{21} \Delta V_0. \quad (S9) $$

Similarly, the zero net current condition for node 6 can be written as the general form with a source term $-G_{67} \Delta V_0$

$$ G_{65} (V_6 - V_5) + G_{67} (V_6 - V_7) = -G_{67} \Delta V_0. \quad (S10) $$

Thus, the zero net flux conditions for the node $i$ connecting their neighbors on the left boundary of the simulation cell introduce a source term $\sum_j G_{ij} \Delta V_0$, while the zero net flux conditions for the node connecting their neighbors on the right boundary introduce a source term $-\sum_j G_{ij} \Delta V_0$.

S2.3. Contact resistance
So far, we have only considered resistance to current of bonded nodes inside one CNT. When two CNTs contact, a larger contact resistance needs to be considered. If nodes $i$ and $k$ form a contact, the zero net current condition of node $i$ should be modified as

$$\sum_j G_{ij} V_i - \sum_j G_{ij} V_j + \sum_k \tilde{G}_{ik} V_i - \sum_k \tilde{G}_{ik} V_k = 0 \ , \ (S11)$$

where $\tilde{G}_{ik}$ is the contact conductance, which is usually much smaller than $G_{ij}$, the conductance between bonded nodes. Similarly, for the contacts formed on the boundary of the simulation cell, source terms need to be added as shown in Eqs. S9 and S10.

S2.4. Governing equation set

Considering the zero net current condition for all the nodes ($i=1$ to $n$), we obtain a governing equation set

$$GV = S \ , \ (S12)$$

with voltage $V = (V_1, V_2, \ldots, V_n)^T$, source term $S = (S_1, S_2, \ldots, S_n)^T$ and the conductance matrix $G = (G_{ij})_{n \times n}$. $S_i$ becomes nonzero if node $i$ connects their neighbors or forms contacts on the left or right boundaries of the simulation cell. $G_{ij}$ becomes nonzero if nodes $i$ and $j$ form a bond or contact. Under the given voltage drop on the boundary of the simulation cell $\Delta V_0$, the conductance between bonded or contacting nodes $G_{ij}$ and $\tilde{G}_{ij}$, and the known structure of the CNT network, we can solve Eq. S12 and obtain the distribution of the voltage in the simulation cell.

Next, in order to calculate the effective conductance of the CNT network, we need to calculate the total current going through the simulation cell. To do so, we can choose an arbitrary cross-section, and the total current across it should be independent of the choice. Here we just take the left boundary of the simulation cell as an example. For a node $i$ connecting its neighboring node $j$ across the left boundary, the current across the boundary can be calculated as

$$I_{ij} = G_{ij} (V_j + \Delta V_0 - V_i) \ . \ (S13)$$

The total flux can then be obtained as the sum of all flux through bonded CNTs $I_{ij}$ and contacts $\bar{I}_{ik}$, $I = \sum_{i,j,k} I_{ij} + \bar{I}_{ik}$. The effective conductance of the matrix can be calculated as $G = I / \Delta V_0$, and the effective resistance as $R = 1 / G$. The electric conductance through
bonded CNTs $G_{ij}$ are related to the CNT resistance as $G_{ij} = 1/R_{node}$, and the electric conductance through contacts $\tilde{G}_{ij}$ are related to the contact resistance as $\tilde{G}_{ij} = 1/R_{contact}$.  

**S3. Analytical model of resistance evolution under loading cycles**

Here we establish an analytical model to relate the resistances in the stretching and transverse directions, $R_x$ and $R_z$ respectively, with the loading history in the limit of very long CNTs, which easily buckles under compression. In this model, we first obtain an expression for the dependence of the mean relative projected length $\xi$ on the strain history, and then establish the relation between $\xi$ and the electrical resistance $R$.

Before stretching, the CNTs are assumed to be straight, with a random distribution of orientation. Therefore, the mean relative projected length $\xi_x = \langle l_x \rangle/h_x$ and $\xi_z = \langle l_z \rangle/h_z$ before stretching are

$$\frac{\xi_x H_x}{l_{CNT}} = \frac{\xi_z H_z}{l_{CNT}} = \frac{2}{\pi} \int_0^{\pi/2} \cos \Theta d\Theta = 0.637,$$  \hspace{1cm} (S14)

where $H_x$ and $H_z$ are the sizes of the simulation cell in $x$ and $z$ directions before the stretching, $l_{CNT}$ is the length of CNTs, and $\Theta$ is the angle between the CNT and the $x$ axis.

After stretching, the CNTs may be curved, and we consider the vector connecting the two end points of every CNT, the end-to-end vector. If the end-to-end vector deforms affinely with the applied strain, it experiences a stretch of $\lambda = 1 + \varepsilon$ in the $x$ direction, and a stretch of $1/\sqrt{\lambda}$ in the $z$ direction. Then the vector of a CNT with the initial orientation $\Theta$ deforms to length $l_{aff}$, and rotates to orientation $\theta$

$$l_{aff} = l_{CNT} \sqrt{\lambda^2 \cos^2 \Theta + \sin^2 \Theta / \lambda} \hspace{1cm}, \hspace{1cm} \theta = \arctan \left( \tan(\Theta) \lambda^{-3/2} \right).$$  \hspace{1cm} (S15)

By setting $l_{aff} = l_{CNT}$, we can obtain a critical angle $\Theta_c = \arccos \left( 1 / (\lambda^2 + \lambda + 1) \right)$. For all CNTs with the initial orientation angle $\Theta > \Theta_c$, the length of the end-to-end vector becomes shorter under the affine deformation $l_{aff} < l_{CNT}$, and therefore we assume that all these CNTs buckle so that the end-to-end vectors will deform affinely with the applied strain. On the other hand, the end-to-end vector will not be able to follow the affine
deformation for all CNTs with the initial orientation angle $\Theta < \Theta_c$, since doing so would require the vector to become longer than the contour length of the CNT. For simplicity, we assume that the end-to-end vector for these CNTs will only rotate to the new orientation specified by the affine deformation, but its length will remain at the contour length of the CNT. Given these assumptions, during loading the mean relative projected length $\xi_x$ and $\xi_z$ can be expressed as

$$\xi_x = \frac{2}{\pi h_x} \left( l_{CNT} \int_0^{\Theta} \cos \theta d\Theta + l_{aff} \int_{\Theta_c}^{\pi/2} \cos \theta d\Theta \right), \quad (S16)$$

$$\xi_z = \frac{2}{\pi h_z} \left( l_{CNT} \int_0^{\Theta} \sin \theta d\Theta + l_{aff} \int_{\Theta_c}^{\pi/2} \sin \theta d\Theta \right), \quad (S17)$$

where the current simulation cell sizes relate the one before stretching by $h_x = \lambda h_x$ and $h_z = H_z / \sqrt{\lambda}$.

During unloading, after reaching the maximal stretch $\lambda_m$, the CNTs with $\Theta > \Theta_c$ reversibly recover from the buckling, and therefore the two ends of the CNTs always deform affinely with the substrate. It can also be proved that CNTs with initial orientation $\Theta < \Theta_c$ buckle during unloading, so the two ends of the CNTs follow the affine deformation of the substrate with the configuration under the maximal stretch $\lambda_m$ as the reference state. During unloading, the mean relative projected length $\xi_x$ and $\xi_z$ can be calculated as

$$\xi_x = \frac{2}{\pi h_x} \left( l_{CNT} \int_0^{\Theta} \cos \theta_m \frac{\lambda}{\lambda_m} d\Theta + l_{aff} \int_{\Theta_c}^{\pi/2} \cos \theta d\Theta \right), \quad (S18)$$

$$\xi_z = \frac{2}{\pi h_z} \left( l_{CNT} \int_0^{\Theta} \sin \theta_m \sqrt{\frac{\lambda_m}{\lambda}} d\Theta + l_{aff} \int_{\Theta_c}^{\pi/2} \sin \theta d\Theta \right), \quad (S19)$$

where $\Theta_m = \arctan \left( \tan(\Theta) \lambda_m^{-3/2} \right)$. Combining Eqs. S16-S19, we obtain Eq. 1 and 2 in the main text.

During the subsequent loading, the CNTs with $\Theta > \Theta_c$ will buckle again, while the CNTs with $\Theta < \Theta_c$ reversibly recover from the buckling until $\lambda_m$, after which Eqs. S16...
and S17 are applicable again. Therefore, the subsequent reloading curve of $\xi - \varepsilon$ overlaps the unloading one until the previous maximal strain is reached.

After obtaining the evolution of $\xi_x$ and $\xi_z$, we further relate them to $R_x$ and $R_z$. The inverse of the mean relative projected length $\eta_x = h_x / \langle l_x \rangle$ represents the least contacts needed for electrons to conduct through the simulation cell in the $x$ direction. Then $N_{CNT}$ number of CNTs can form $N_{CNT} / \eta_x$ parallel paths of conduction, so the resistance $R_x$ should be proportional to $\eta_x^2 / N_{CNT}$. When the contact resistance $R_{contact}$ is much higher than the resistance of the CNTs, the resistance in the stretching direction can be estimated as

$$R_x = \alpha R_{contact} \eta_x^2 / N_{CNT}, \quad (S20)$$

where $\alpha$ is a constant on the order of 1 related to the morphology of the CNT network. Similarly, the resistance in the $z$ direction can be calculated as

$$R_z = \alpha R_{contact} \eta_z^2 / N_{CNT}. \quad (S21)$$

Combining Eqs. S16-S21, we can analytically calculate the evolution of the resistance $R_x$ and $R_z$ with respect to an arbitrary loading history.
4. Supplementary figures

Fig. S1. Experimental setup for the in situ measurement of the resistance of the CNT thin film under a cyclic loading. (A) Images with no strain applied ($\varepsilon = 0$), and (B) with $\varepsilon = 0.6$. Analysis of the images (A) and (B) shows that the width and length changes of the PDMS substrate follow the one of an incompressible material under uniaxial deformation, and that the deformation of the CNT film is the same as the PDMS substrate.
Fig. S2. Example of CNTs sliding in the same bundles as strain increases.
Fig. S3. Effect of the simulation cell size on the convergence of the simulation. The resistance of the CNT network increases and then decreases during a loading and unloading cycle of maximal strain 0.4, forming a hysteresis. Each CNT has length \( l_{\text{CNT}} = 800 \text{ nm} \) and diameter \( d_{\text{CNT}} = 1 \text{ nm} \), and is discretized by \( N_{\text{node}} = 40 \) nodes. The simulation cell has size \( H_x \) and \( H_z \) in the \( x \) (loading) and \( z \) (transverse) directions, and 100 nm in the \( y \) direction. The cell size is varied from (A) \( H_x = H_z = 400 \text{ nm} \), (B) \( H_x = H_z = 800 \text{ nm} \), to (C) \( H_x = H_z = 1600 \text{ nm} \). The density of the CNTs is fixed, and the number of CNTs is (A) \( N_{\text{CNT}} = 30 \), (B) \( N_{\text{CNT}} = 120 \) and (C) \( N_{\text{CNT}} = 480 \) respectively. All the results shown here are the average of around 10 simulations. As we can see, even when the cell size is as small as half of the CNT length, the cell size almost does not affect the calculated resistance change.
Fig. S4. Effect of the contact radius $r_{\text{contact}}$ on the convergence of the simulation, (A) $r_{\text{contact}} = 0.2$ nm, (B) $r_{\text{contact}} = 0.4$ nm, (C) $r_{\text{contact}} = 0.6$ nm. The CNT network is composed of $N_{\text{CNT}} = 480$ CNTs, each with length $l_{\text{CNT}} = 800$ nm, diameter $d_{\text{CNT}} = 1$ nm, and discretized by $N_{\text{node}} = 40$ nodes. As we can see, the resistance change is not sensitive to $r_{\text{contact}}$. In the following simulations, we always use $r_{\text{contact}} = 0.4$ nm.
Fig. S5. A typical simulation result of the relative resistance change in the (A) $x$ (loading) and (B) $z$ (transverse) directions under three sequentially increasing strain cycles of 20%, 40% and 60% without averaging, for $N_{CNT} = 135$ CNTs with length $l_{CNT} = 2400$ nm, diameter $d_{CNT} = 1$ nm, each discretized by $N_{node} = 120$ nodes, in a simulation cell with $H_x = H_z = 1200$ nm. The contact resistance is set as $R_{contact} = 200$ k$\Omega$, and the resistance of a single CNT is $R_{CNT} = 17.3$ k$\Omega$. 
Fig. S6. Direct comparison between the CGMS simulation and the experimental results of the relative resistance change in the (A) x (loading) and (B) z (transverse) directions under three sequentially increasing strain cycles of 20%, 40% and 60%. The experimental and CGMS simulation results are shown in Fig. 2A, B and Fig. 2C, D respectively.
Fig. S7. The relative sheet resistance change in the (A) x and (B) z directions under three sequentially increasing strain cycles of 20%, 40% and 60%, for $N_{\text{CNT}} = 135$ CNTs with length $l_{\text{CNT}} = 2400$ nm, diameter $d_{\text{CNT}} = 1$ nm, each discretized by $N_{\text{node}} = 120$ nodes, in a simulation cell with $H_x = H_z = 1200$ nm. Sheet resistance is defined as $R_{\text{x}} = R_{x}h_{x}/h_{x}$ and $R_{\text{z}} = R_{z}h_{z}/h_{z}$. The contact resistance is set as $R_{\text{contact}} = 200$ k$\Omega$, and the resistance of a single CNT is $R_{\text{CNT}} = 17.3$ k$\Omega$. 
Fig. S8. Evolution of the mean angle difference between neighboring CNT segments $\langle \Delta \theta_{\text{node}} \rangle$ under three sequentially increasing strain cycles of 20%, 40% and 60%.
Fig. S9. Evolution of the number of total contacts $N_{contact}$ between CNTs under three sequentially increasing strain cycles of 20%, 40% and 60%.
Fig. S10. The mean projected lengths of the end-to-end vectors of CNTs in the $x$ and $z$ directions normalized by the original simulation cell size (A, B) and the current cell size (C, D).
Fig. S11. The correlation of the relative resistance changes in the x direction $\Delta R_x / R_{x0}$ (A) and z direction $\Delta R_z / R_{z0}$ (B) with the relative changes of the inverse mean projected lengths in the x and z directions $\Delta \eta_x / \eta_{x0}$ and $\Delta \eta_z / \eta_{z0}$ respectively.
Fig. S12. The evolution of the inverse mean relative projected lengths in the $x$ and $z$ directions $\eta_x$ (A, C) and $\eta_z$ (B, D) by CGMS simulation (A, B) and analytical modeling (C, D). The simulation and analytical results show very good agreement.
Fig. S13. Effect of CNT density on the evolution of the resistance change, (A) $N_{CNT} = 30$ nanotubes, (B) $N_{CNT} = 60$, (C) $N_{CNT} = 90$. For all cases, the CNTs have length $l_{CNT} = 1600$ nm, diameter $d_{CNT} = 1$ nm, each discretized by $N_{node} = 80$ nodes, in a simulation cell with $H_x = H_z = 800$ nm.
Fig. S14. Effect of the CNT diameter on the resistance change. (A) $d_{\text{CNT}} = 1\,\text{nm}$, (B) $d_{\text{CNT}} = 1.4\,\text{nm}$ and (C) $d_{\text{CNT}} = 2\,\text{nm}$. For all cases, the CNT network is composed of $N_{\text{CNT}} = 60$ CNTs, each with length $l_{\text{CNT}} = 1600\,\text{nm}$, discretized by $N_{\text{node}} = 80$ nodes.
Movie S1. The evolution of the morphology of a CNT network under three sequentially increasing strain cycles of 20%, 40% and 60%. The CNT network is composed of $N_{CNT} = 135$ CNTs with length $l_{CNT} = 2400$ nm, diameter $d_{CNT} = 1$ nm, each discretized by $N_{node} = 120$ nodes, in a simulation cell with $H_x = H_z = 1200$ nm.

References