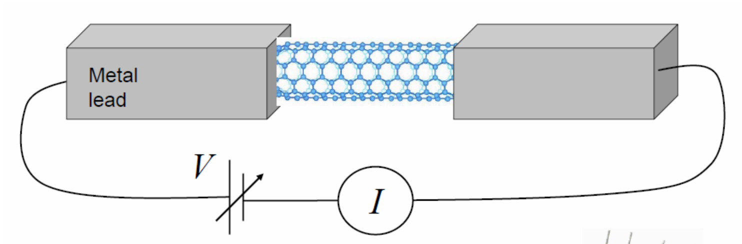
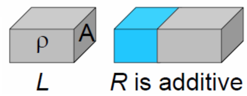
"Ideal" Electrical Resistance in 1-D



- Ohm's Law: R = V/I [Ω]
- Bulk materials, resistivity ρ: R = ρL/A



- Nanoscale systems (*coherent* transport)
  - R (G = 1/R) is a global quantity
  - R cannot be decomposed into subparts, or added up from pieces

Charge & Energy Current Flow <u>in 1-D</u>

- Remember (net) current $J_x \approx x \times n \times v$ where $x = e$ or E

$$J_e = e\sum_k n_k \mathbf{v}_k = e\sum_k [g_k f_k]\mathbf{v}_k \rightarrow J_e = e\int_k g_k f_k' \mathbf{v}_k dk$$

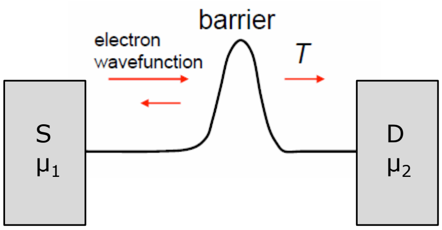
Net contribution

$$J_E = \sum_k E_k n_k \mathbf{v}_k = \sum_k E_k [g_k f_k]\mathbf{v}_k \rightarrow J_E = \int_k E_k g_k f_k \mathbf{v}_k dk$$

- Let's focus on charge current flow, for now
- Convert to integral over energy, use Fermi distribution

Conductance as Transmission

$$I = \frac{2e}{h}\int f(E)T(E)dE$$

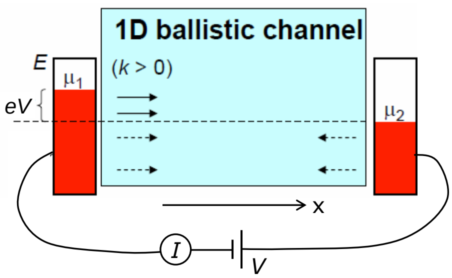


$$J_e = e\int_k g_k f_k T_k \mathbf{v}_k dk$$

Rolf Landauer (1927-1999)

- Two terminals (S and D) with Fermi levels μ_1 and μ_2
- S and D are big, ideal electron reservoirs, MANY k-modes
- Transmission channel has only ONE mode, M = 1

Conductance of 1-D Quantum Wire



$$I = e\int_k (g_k f_k) T_k v_k dk \qquad v = \frac{1}{\hbar}\frac{dE}{dk}$$

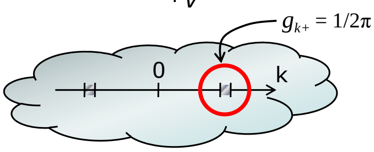
$$I = \frac{e}{h}\int_{\mu_2}^{\mu_1} f_1(E)T(E)dE$$

x2 spin

$$G = \frac{I}{V} = \frac{e^2}{h}$$

quantum of electrical conductance (per spin per mode)

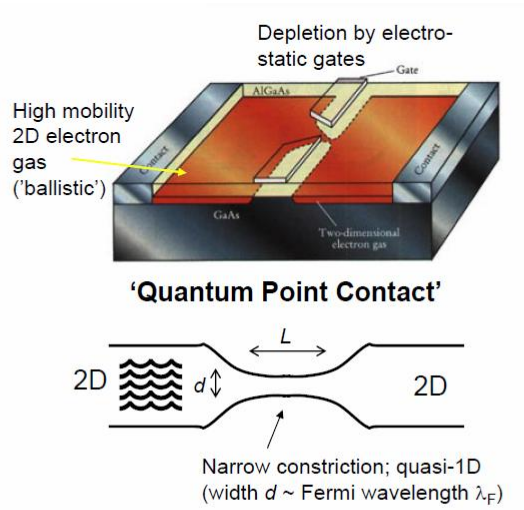
1D k-space

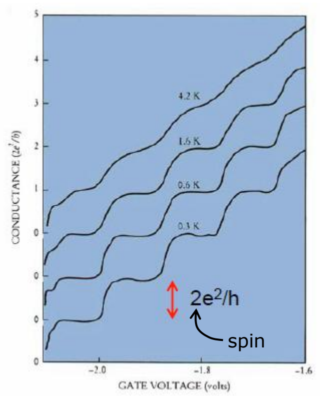


- Voltage applied is Fermi level separation: $eV = \mu_1 - \mu_2$
- Channel = 1D, ballistic, coherent, no scattering (T=1)

Quasi-1D Channel in 2D Structure

van Wees, *Phys. Rev. Lett.* (1988)





Limited conductance $2e^2/h$ even without scattering, regardless of length *L*: "contact resistance"

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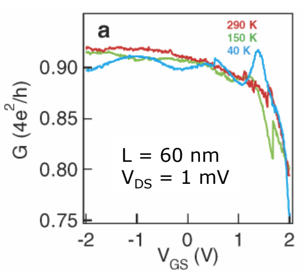
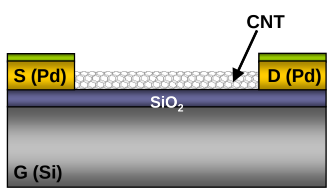
Quantum Conductance in Nanotubes

- 2x subbands in nanotubes, and 2x from spin
- "Best" conductance of $4e^2/h$, or lowest R = 6,453 Ω
- In practice we measure higher resistance, due to scattering, defects, imperfect contacts (Schottky barriers)





Javey et al., *Phys. Rev. Lett.* (2004)

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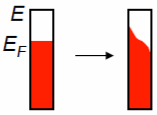
Finite Temperatures

- Electrons in leads according to Fermi-Dirac distribution



$$f(E, E_F) = \frac{1}{\exp\left(\frac{E-E_F}{k_B T}\right) + 1}$$

- Conductance with n channels, at finite temperature T:

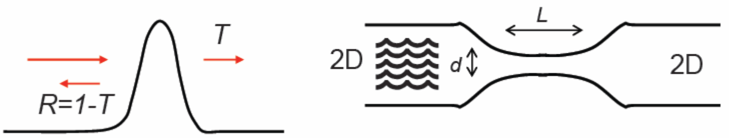
$$G(E_F, T) = \frac{2e^2}{h} \sum_n \int T_n(E) \left(-\frac{df}{dE}\right) dE \approx \frac{2e^2}{h} \sum_n f(E_n - E_F)$$

- At even higher T: "usual" incoherent transport (dephasing due to inelastic scattering, phonons, etc.)

Where Is the Resistance?

S. Datta, "Electronic Transport in Mesoscopic Systems" (1995)

One-channel case again:



$$G = \frac{2e^2}{h} T \quad \text{(transmission probability } T\text{)}$$

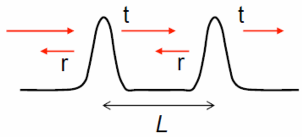
scattering from barriers (zero for perfect conductor)

$$\text{Resistance} = \frac{h}{2e^2}\frac{1}{T} = \frac{h}{2e^2}\left(1 + \frac{1-T}{T}\right) = \boxed{\frac{h}{2e^2}} + \boxed{\frac{h}{2e^2}\frac{R}{T}} \quad (T + R = 1)$$

quantized contact resistance

Multiple Barriers, <u>Coherent</u> Transport

Two identical barriers in series:

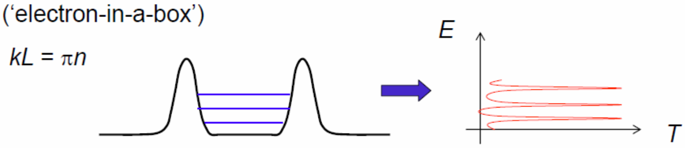


- Coherent, resonant transport
- $L < L_\Phi$ (phase-breaking length); electron is truly a wave

- Perfect transmission through resonant, quasi-bound states:

Quasi-bound states:
('electron-in-a-box')

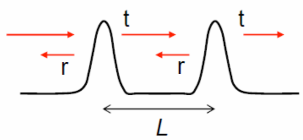
Resonant transmission:



Spectrum
(0D system)

Transport measurement

Multiple Barriers, <u>Incoherent</u> Transport



- $L > L_\Phi$ (phase-breaking length); electron phase gets randomized at, or between scattering sites

- Total transmission (no interference term):

$$T_{\text{total}} = \frac{|t_1|^2 |t_2|^2}{1 - |r_1|^2 |r_2|^2}$$

- Resistance (scatterers in series):

$$\text{Resistance} = \frac{h}{2e^2}\left(1 + \frac{|r_1|^2}{|t_1|^2} + \frac{|r_2|^2}{|t_2|^2} + \cdots\right) = \frac{h}{2e^2}\left(1 + \frac{L}{\Lambda}\right)$$

average mean free path; remember Matthiessen's rule!

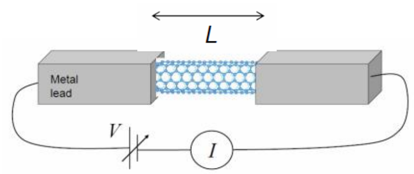
- Ohmic addition of resistances from independent scatterers

Where Is the Power (I^2R) Dissipated?

- Consider, e.g., a single nanotube
- <u>Case I</u>: **L << Λ**

  R ~ $h/4e^2$ ~ 6.5 kΩ

  Power I^2R → ?



- <u>Case II</u>: **L >> Λ**

  R ~ $h/4e^2(1 + L/\Lambda)$

  Power I^2R → ?

- Remember $\frac{1}{\Lambda} \approx \frac{1}{\Lambda_{op.phon.}} + \frac{1}{\Lambda_{ac.phon.}} + \frac{1}{\Lambda_{imp.}} + \frac{1}{\Lambda_{def.}} + \frac{1}{\Lambda_{e-e}}$

1D Wiedemann-Franz Law (WFL)

- Does the WFL hold in 1D? → <u>YES</u>
- 1D ballistic electrons carry energy too, what is their equivalent thermal conductance?

$$G_{th} = L\sigma_e T = \left(\frac{\pi^2 k_B^2}{3e^2}\right)\left(\frac{e^2}{h}\right)T = \boxed{\frac{\pi^2 k_B^2 T}{3h}}$$

(x2 if electron spin included)

$$G_{th} \approx 0.28 \text{ nW/K at } 300 \text{ K}$$

VOLUME 78, NUMBER 6 PHYSICAL REVIEW LETTERS 10 FEBRUARY 1997

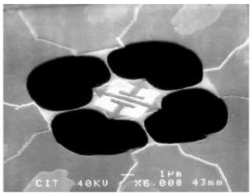
Thermal Conductivity and Lorenz Number for One-Dimensional Ballistic Transport

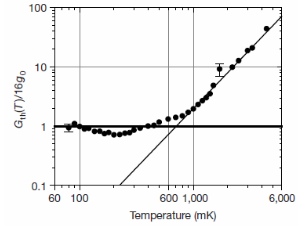
We study the thermal conductivity and Lorenz number of charge carriers for one-dimensional ballistic transport within the correlation function formalism. The carrier transit time between two ideal contacts is found to substitute for the collision time in the definition of a ballistic thermal conductivity. A universal thermal conductance $K = 2\pi^2 k_B^2 T/3h$ is naturally obtained for the degenerate case.

Greiner, Phys. Rev. Lett. (1997)

Phonon Quantum Thermal Conductance

- <u>Same</u> thermal conductance quantum, irrespective of the carrier statistics (Fermi-Dirac vs. Bose-Einstein)

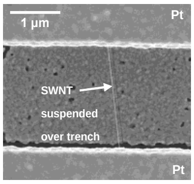


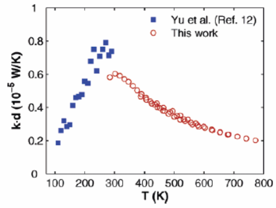


Phonon G_{th} measurement in GaAs bridge at T < 1 K

Schwab, *Nature* (2000)

$$G_{th} = \frac{\pi^2 k_B^2 T}{3h} \approx 0.28 \text{ nW/K at 300 K}$$





Single nanotube G_{th}=2.4 nW/K at T=300K

Pop, *Nano Lett.* (2006)

Matlab tip:

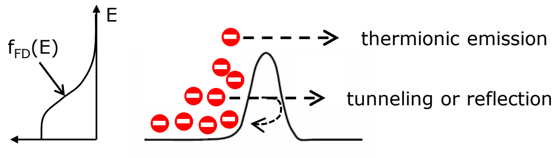
```
>> syms x;
>> int(x^2*exp(x)/(exp(x)+1)^2,0,Inf)
ans =
1/6*pi^2
```


Electrical vs. Thermal Conductance G_0

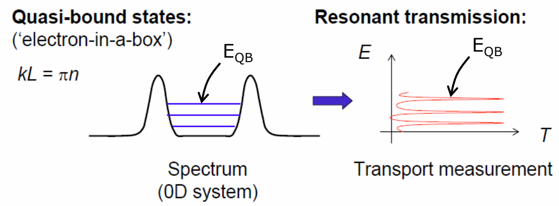
- Electrical experiments → steps in the conductance (not observed in thermal experiments)
- In electrical experiments the chemical potential (Fermi level) and temperature can be independently varied
  - Consequently, at low-T the sharp edge of the Fermi-Dirac function can be swept through 1-D modes
  - Electrical (electron) conductance quantum: $G_0 = dI_e/dV$
- In thermal (phonon) experiments only the temperature can be swept
  - The broader Bose-Einstein distribution smears out all features except the lowest lying modes at low temperatures
  - Thermal (phonon) conductance quantum: $G_0 = dQ_{th}/dT$

Back to the Quantum-Coherent Regime

- Single energy barrier – how do you get across?

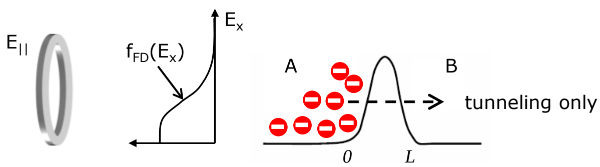


- Double barrier: transmission through quasi-bound (QB) states



- Generally, need $\lambda \sim L \leq L_{\Phi}$ (phase-breaking length)

Wentzel-Kramers-Brillouin (WKB)



- Assume smoothly varying potential barrier, no reflections

$$T = \frac{|\psi_{trans.}|^2}{|\psi_{incid.}|^2} \approx \exp\left(-2\int_0^L |k(x)|\, dx\right)$$

k(x) depends on energy dispersion

$$J_{A\to B} = (\#\text{incident states}) \int f_A g_A T(E_x)(1-f_B) g_B dE$$

E.g. in 3D, the net current is:

$$J = J_{A\to B} - J_{B\to A} = \frac{qm^*}{2\pi^2\hbar^3}\int (f_A - f_B) g_A g_B T(E) dE$$

Fancier version of Landauer formula!

Band-to-Band Tunneling

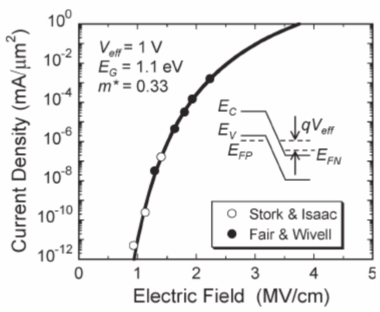
- Assuming parabolic energy dispersion $E(k) = \hbar^2 k^2/2m^*$

$$T(E_x) \approx \exp\left(-\frac{4\sqrt{2m_x^*}E_x^{3/2}}{3q\hbar F}\right)$$

F = electric field

- E.g. band-to-band (Zener) tunneling in silicon diode

$$J_{BB} = \frac{q^3 F V_{eff}}{4\pi^3\hbar^2}\sqrt{\frac{2m^*}{E_G}}\exp\left(-\frac{4\sqrt{2m_x^*}E_G^{3/2}}{3q\hbar F}\right)$$



See, e.g. Kane, *J. Appl. Phys.* 32, 83 (1961)