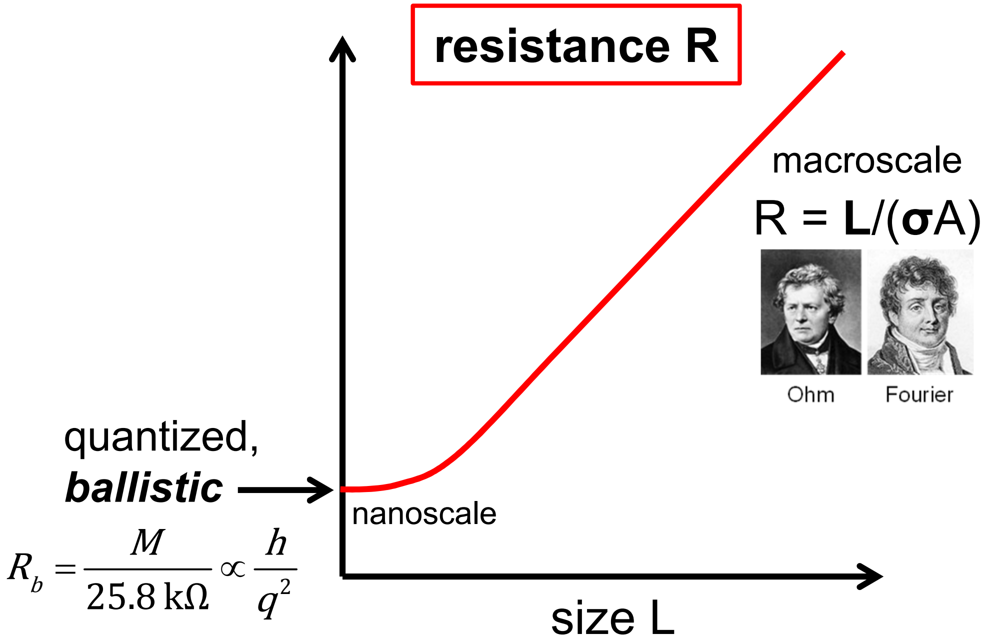
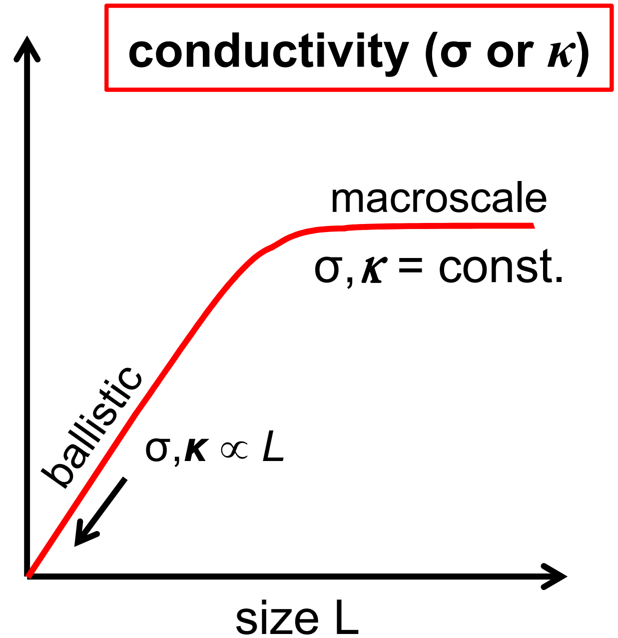
Conductance Quantization

- One-dimensional ballistic/coherent transport
- Landauer theory
- The role of contacts
- Quantum of electrical and thermal conductance
- One-dimensional Wiedemann-Franz law

“Ideal” Electrical Resistance in 1-D





- Macroscale, R is additive: 1 + 1 = 2
- ***Nanoscale***, R is quantized: 1 + 1 = 1
  - Occurs when system size is comparable to the electron or phonon (heat) wavelengths and collision distance (10-100 nm)
  - Both electrical and thermal resistance can be **quasi-ballistic**

Charge & Energy Current Flow <u>in 1-D</u>

- Remember (net) current $\boldsymbol{J_x} \approx \boldsymbol{x}\times\boldsymbol{n}\times\boldsymbol{v}$ where $\boldsymbol{x} = \boldsymbol{q}$ or $\boldsymbol{E}$

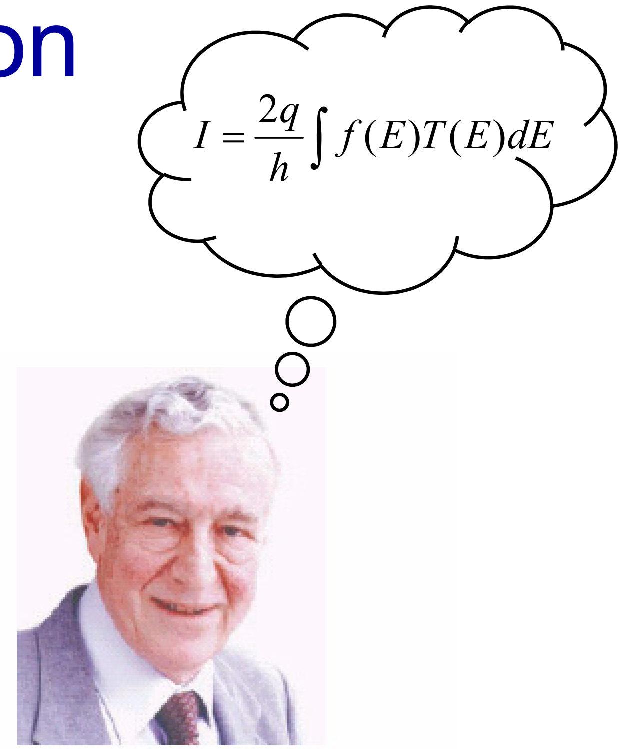
Net contribution

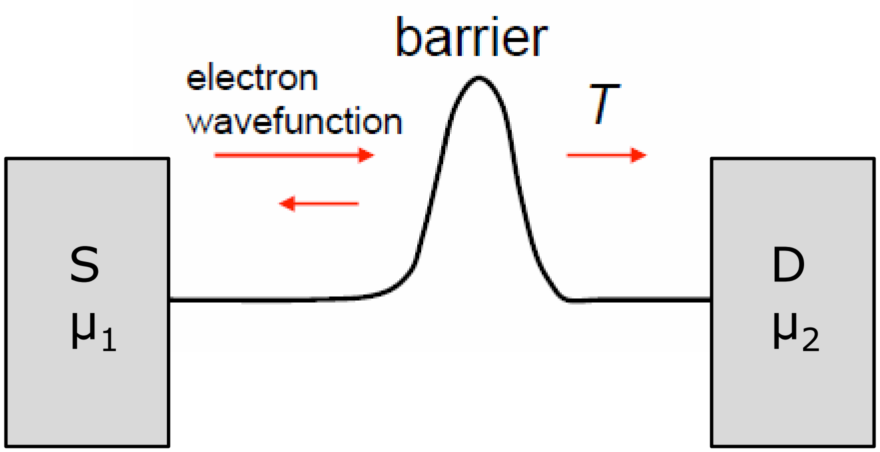
$$J_q = q\sum_k n_k \mathbf{v}_k = q\sum_k [g_k f_k]\mathbf{v}_k \quad\rightarrow\quad J_q = q\int_k g_k f_k' \mathbf{v}_k dk$$

$$J_E = \sum_k E_k n_k \mathbf{v}_k = \sum_k E_k [g_k f_k]\mathbf{v}_k \quad\rightarrow\quad J_E = \int_k E_k g_k f_k \mathbf{v}_k dk$$

- Let's focus on charge current flow, for now
- Convert to integral over energy, use Fermi distribution

Conductance as Transmission



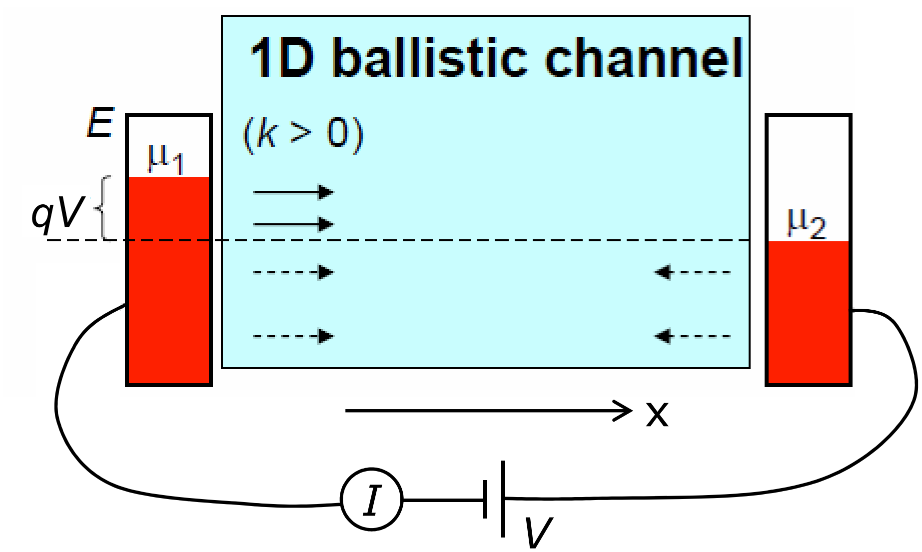


$$J_q = q\int_k g_k f_k T_k \mathbf{v}_k dk$$

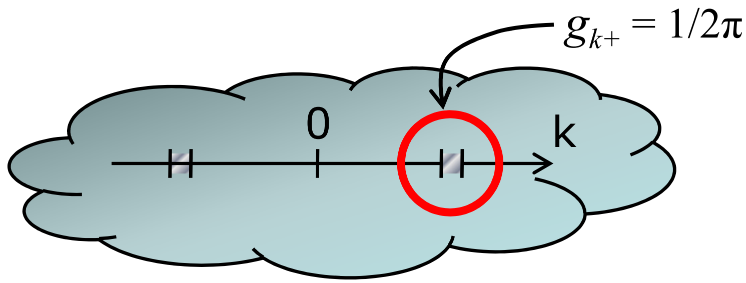
Rolf Landauer (1927-1999)

- Two terminals (S and D) with Fermi levels μ_1 and μ_2
- S and D are big, ideal electron reservoirs, MANY k-modes
- Transmission channel has only ONE mode, M = 1

Conductance of 1-D Quantum Wire



1D k-space



$$I = q\int_k \left(g_k f_k\right) T_k v_k \, dk$$

$$v = \frac{1}{\hbar}\frac{dE}{dk}$$

$$I = \frac{q}{h}\int_{\mu_2}^{\mu_1} f_1(E) T(E) \, dE$$

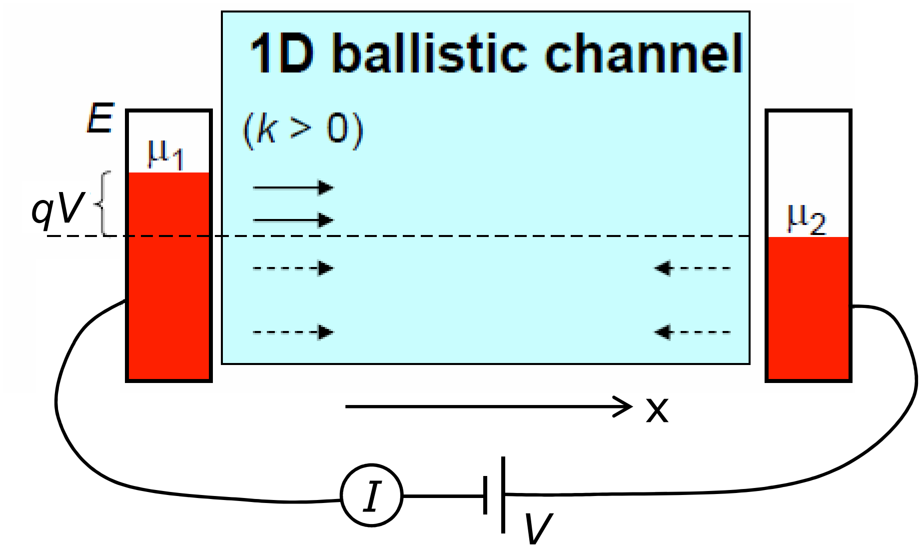
x2 spin

$$G = \frac{I}{V} = \frac{q^2}{h}$$

quantum of electrical conductance (per spin per mode)

- Voltage applied is Fermi level separation: $qV = \mu_1 - \mu_2$
- Channel = 1D, ballistic, coherent, no scattering (T=1)

Simple "Proof" by Uncertainty Principle



Current & voltage in quantum channel are

$$I = \frac{q}{\tau} \qquad V = \frac{E}{q}$$

where τ is the transit time.

Thus, by Heisenberg Uncertainty Principle:

$$G = \frac{I}{V} = \frac{q^2}{E\tau} = \frac{q^2}{h} \approx \frac{1}{25.8\,\text{k}\Omega}$$

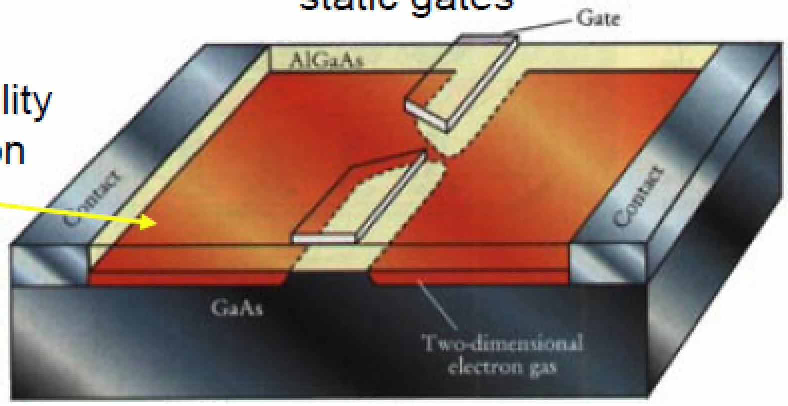
- Note this is for one electron, one (ballistic) quantum channel
- Real resistors can have more quantum channels (M)
- Conversely, scattering can increase resistance (T < 1)

Quasi-1D Channel in 2D Structure

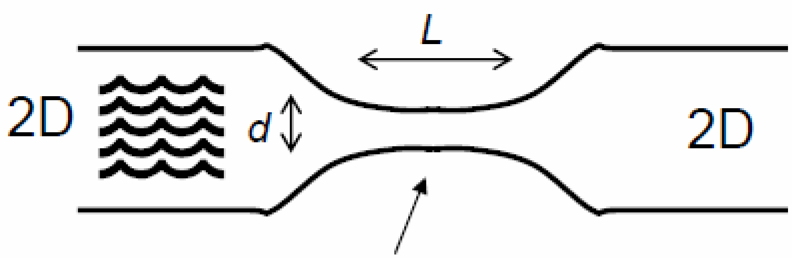
Van Wees et al., *Phys. Rev. Lett*. 60, 848 (1988)

Depletion by electro-
static gates

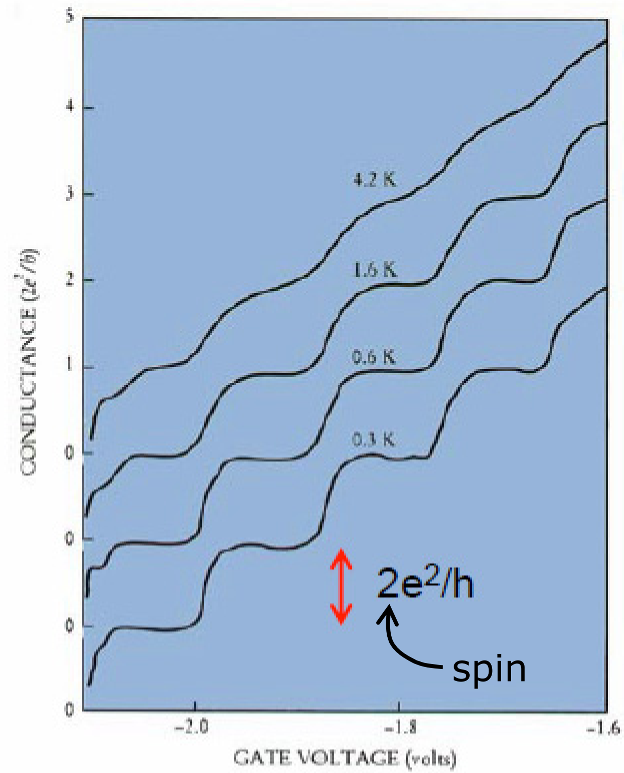
High mobility
2D electron
gas
('ballistic')



'Quantum Point Contact'



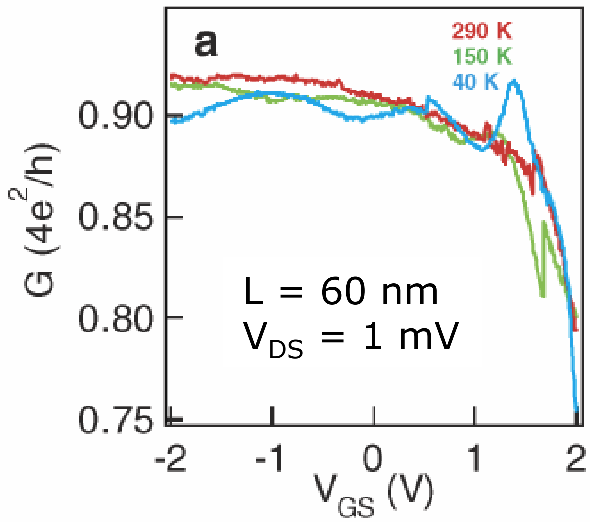
Narrow constriction; quasi-1D
(width d ~ Fermi wavelength λ_F)

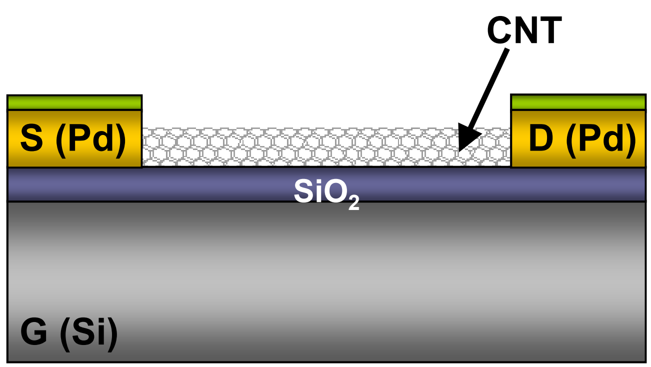


Limited conductance $2e^2/h$
even without scattering,
regardless of length L:
"contact resistance"

Quantum Conductance in Nanotubes

- 2x sub-bands in nanotubes, and 2x from spin
- "Best" conductance of $4q^2/h$, or lowest R = 6,453 Ω
- In practice we measure higher resistance, due to scattering, defects, imperfect contacts (Schottky barriers)

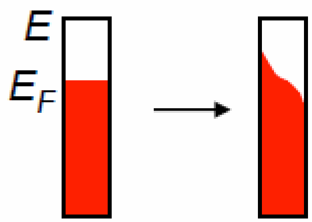




Javey et al., *Phys. Rev. Lett.* (2004)

Finite Temperatures

- Electrons in leads according to Fermi-Dirac distribution



$$f(E, E_F) = \frac{1}{\exp\left(\frac{E - E_F}{k_B T}\right) + 1}$$

- Conductance with n channels, at finite temperature T:

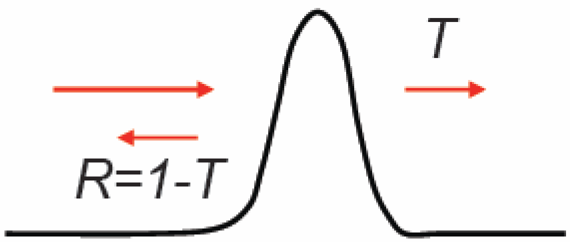
$$G(E_F, T) = \frac{2e^2}{h} \sum_n \int T_n(E) \left(-\frac{df}{dE}\right) dE \approx \frac{2e^2}{h} \sum_n f(E_n - E_F)$$

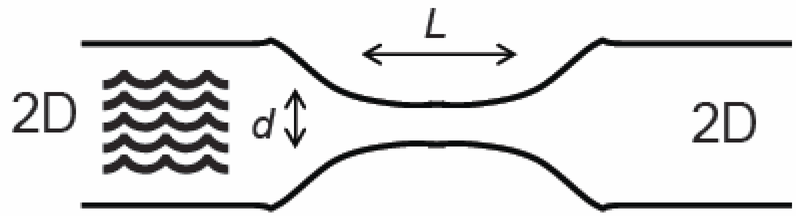
- At even higher T: “usual” incoherent transport (dephasing due to inelastic scattering, phonons, etc.)

Where Is the Resistance?

S. Datta, “Electronic Transport in Mesoscopic Systems” (1995)

One-channel case again:





$$G = \frac{2e^2}{h} T \quad (\text{transmission probability } T)$$

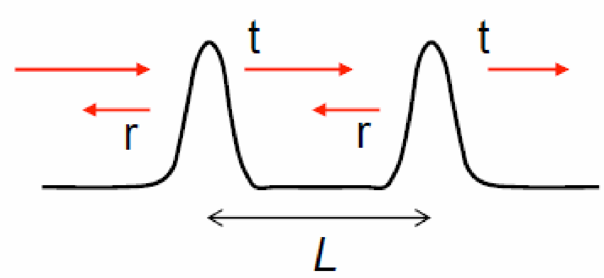
scattering from barriers (zero for perfect conductor)

$$\text{Resistance} = \frac{h}{2e^2}\frac{1}{T} = \frac{h}{2e^2}\left(1 + \frac{1-T}{T}\right) = \boxed{\frac{h}{2e^2}} + \boxed{\frac{h}{2e^2}\frac{R}{T}} \quad (T + R = 1)$$

quantized contact resistance

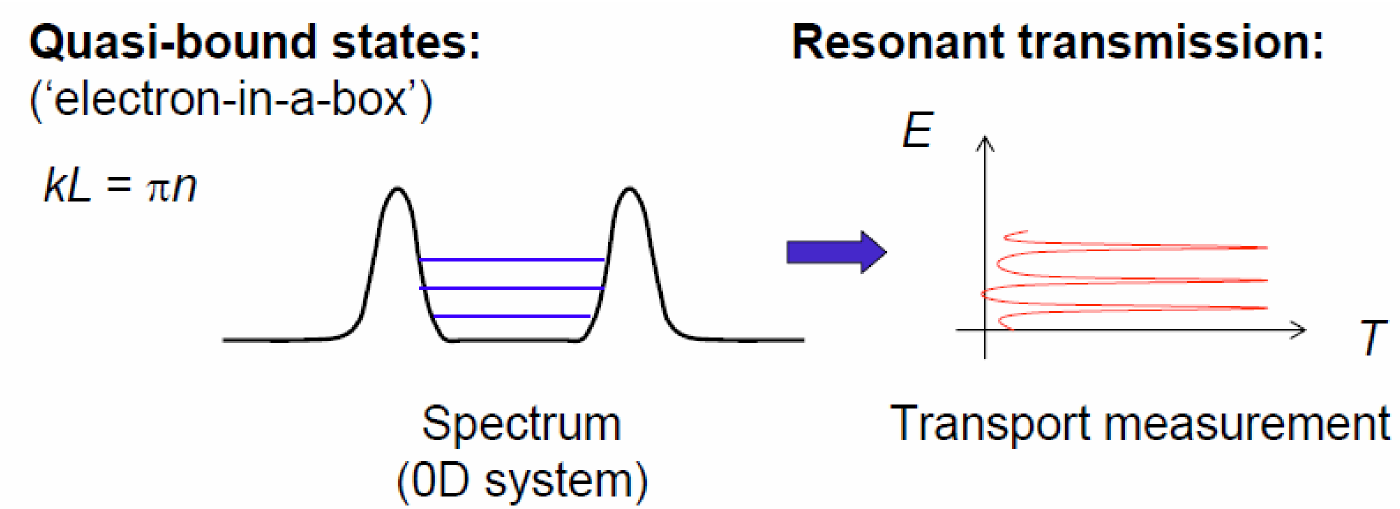
Multiple Barriers, Coherent Transport

Two identical barriers in series:

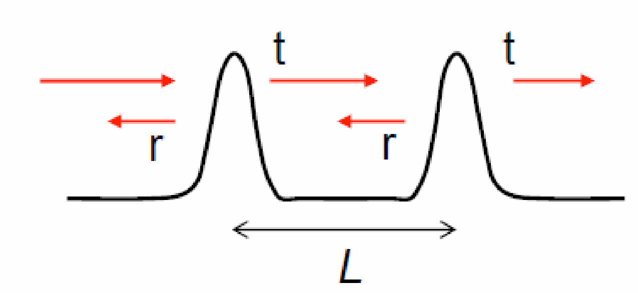


- Coherent, resonant transport
- $L < L_{\Phi}$ (phase-breaking length); electron is truly a wave

- Perfect transmission through resonant, quasi-bound states:



Multiple Barriers, Incoherent Transport



- $L > L_\Phi$ (phase-breaking length); electron phase gets randomized at, or between scattering sites

- Total transmission (no interference term):

$$T_{\text{total}} = \frac{|t_1|^2 |t_2|^2}{1 - |r_1|^2 |r_2|^2}$$

- Resistance (scatterers in series):

$$\text{Resistance} = \frac{h}{2e^2}\left(1 + \frac{|r_1|^2}{|t_1|^2} + \frac{|r_2|^2}{|t_2|^2} + \cdots\right) = \frac{h}{2e^2}\left(1 + \frac{L}{\Lambda}\right)$$

average mean free path; remember Matthiessen's rule!

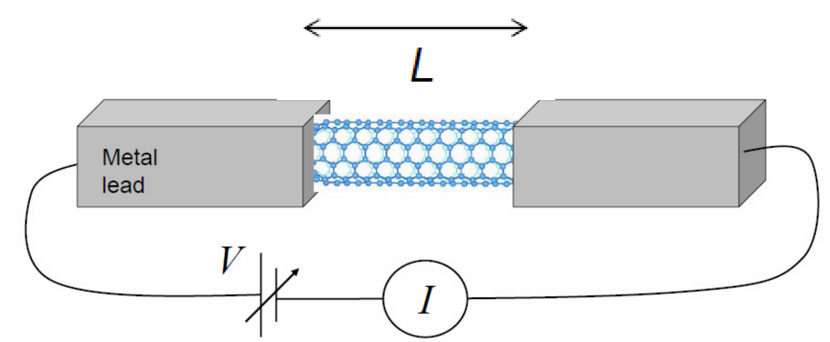
- Ohmic addition of resistances from independent scatterers

Where Is the Power (I^2R) Dissipated?

- Consider, e.g., a single nanotube
- Case I: $\mathbf{L} \ll \mathbf{\Lambda}$

  $R \sim h/4e^2 \sim 6.5\ k\Omega$

  Power $I^2R \rightarrow$?



- Case II: $\mathbf{L} \gg \mathbf{\Lambda}$

  $R \sim h/4e^2(1 + \mathbf{L}/\mathbf{\Lambda})$

  Power $I^2R \rightarrow$?

- Remember $$\frac{1}{\Lambda} \approx \frac{1}{\Lambda_{op.phon.}} + \frac{1}{\Lambda_{ac.phon.}} + \frac{1}{\Lambda_{imp.}} + \frac{1}{\Lambda_{def.}} + \frac{1}{\Lambda_{e-e}}$$

1D Wiedemann-Franz Law (WFL)

- Does the WFL hold in 1D? → YES
- 1D ballistic electrons carry energy too, what is their equivalent thermal conductance?

$$G_{th} = L\sigma_e T = \left(\frac{\pi^2 k_B^2}{3e^2}\right)\left(\frac{e^2}{h}\right) T = \boxed{\frac{\pi^2 k_B^2 T}{3h}}$$

(x2 if electron spin included)

$$G_{th} \approx 0.28 \ \text{nW/K at } 300 \text{ K}$$

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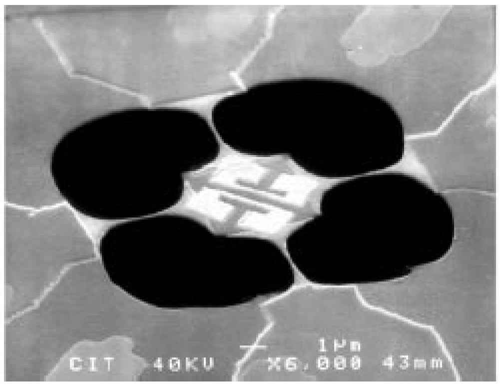
Thermal Conductivity and Lorenz Number for One-Dimensional Ballistic Transport

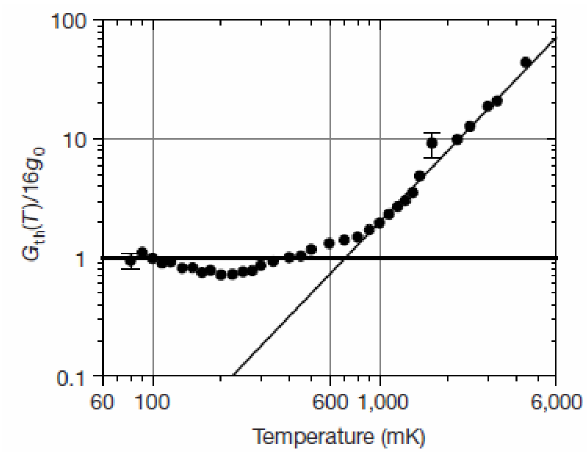
We study the thermal conductivity and Lorenz number of charge carriers for one-dimensional ballistic transport within the correlation function formalism. The carrier transit time between two ideal contacts is found to substitute for the collision time in the definition of a ballistic thermal conductivity. A universal thermal conductance $K = 2\pi^2 k_B^2 T/3h$ is naturally obtained for the degenerate case.

Greiner, Phys. Rev. Lett. (1997)

Phonon Quantum Thermal Conductance

- Same thermal conductance quantum, irrespective of the carrier statistics (Fermi-Dirac vs. Bose-Einstein)

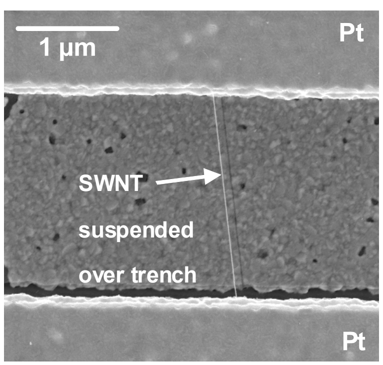


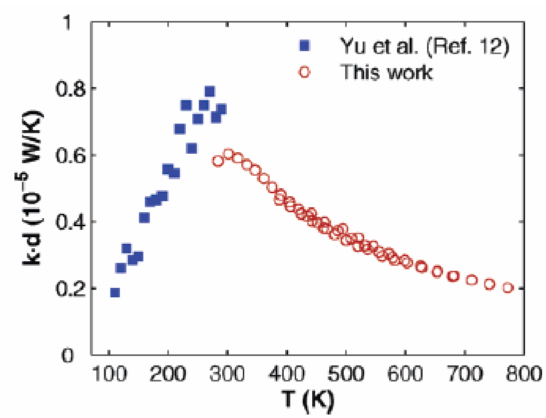


Phonon G_{th} measurement in GaAs bridge at T < 1 K

Schwab, *Nature* (2000)

$$G_{th} = \frac{\pi^2 k_B^2 T}{3h} \approx 0.28 \text{ nW/K at 300 K}$$





Single nanotube G_{th}=2.4 nW/K at T=300K

Pop, *Nano Lett.* (2006)

Matlab tip:

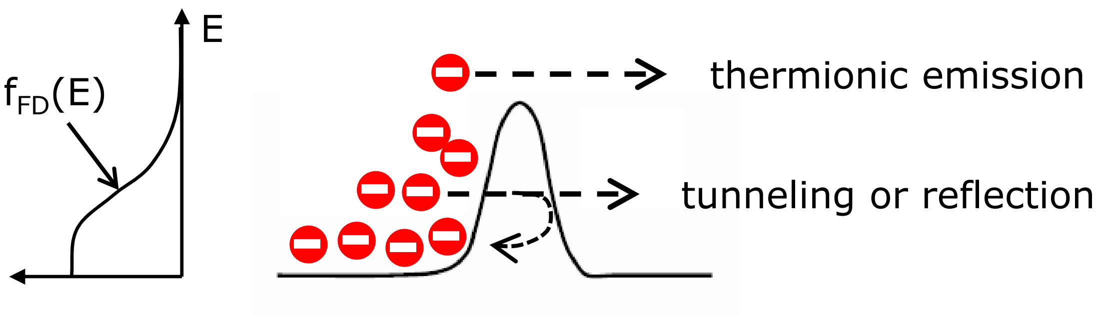
```
>> syms x;
>> int(x^2*exp(x)/(exp(x)+1)^2,0,Inf)
ans =
1/6*pi^2
```

Electrical vs. Thermal Conductance G_0

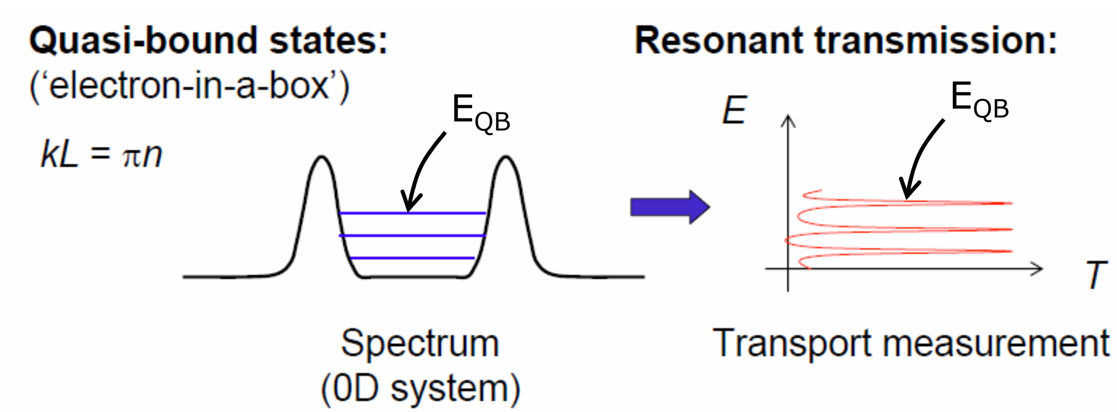
- Electrical experiments → steps in the conductance (not observed in thermal experiments)
- In electrical experiments the chemical potential (Fermi level) and temperature can be independently varied
  – Consequently, at low-T the sharp edge of the Fermi-Dirac function can be swept through 1-D modes
  – Electrical (electron) conductance quantum: $G_0 = (dI_e/dV)|_{low\ dV}$
- In thermal (phonon) experiments only the temperature can be swept
  – The broader Bose-Einstein distribution smears out all features except the lowest lying modes at low temperatures
  – Thermal (phonon) conductance quantum: $G_0 = (dQ_{th}/dT)\ |_{low\ dT}$

Back to the Quantum-Coherent Regime

- Single energy barrier – how do you get across?

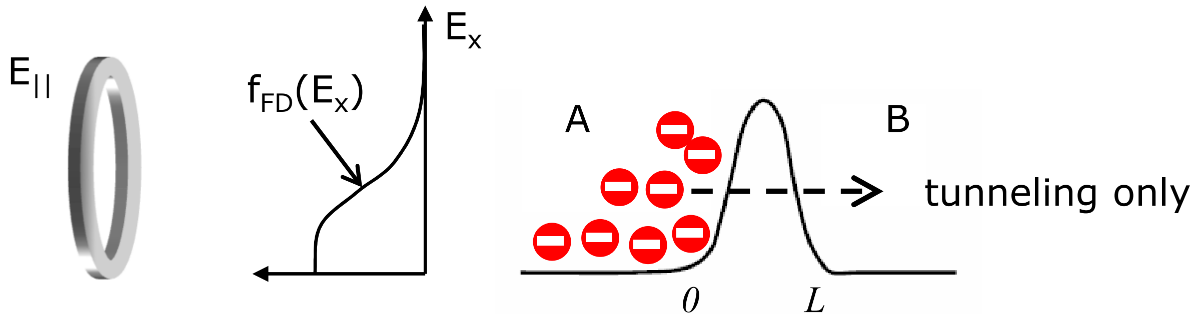


- Double barrier: transmission through quasi-bound (QB) states



- Generally, need $\lambda \sim L \leq L_{\Phi}$ (phase-breaking length)

Wentzel-Kramers-Brillouin (WKB)



- Assume smoothly varying potential barrier, no reflections

$$T = \frac{|\psi_{trans.}|^2}{|\psi_{incid.}|^2} \approx \exp\left(-2\int_0^L |k(x)|\, dx \right)$$

k(x) depends on energy dispersion

$$J_{A\to B} = (\#\text{incident states}) \int f_A g_A T(E_x)(1 - f_B) g_B dE$$

E.g. in 3D, the <u>net</u> current is:

$$J = J_{A\to B} - J_{B\to A} = \frac{qm^*}{2\pi^2\hbar^3}\int (f_A - f_B) g_A g_B T(E) dE$$

Fancier version of Landauer formula!