Thermal Resistance (measurements & simulations) In Electronic Devices

(part of an online short course)

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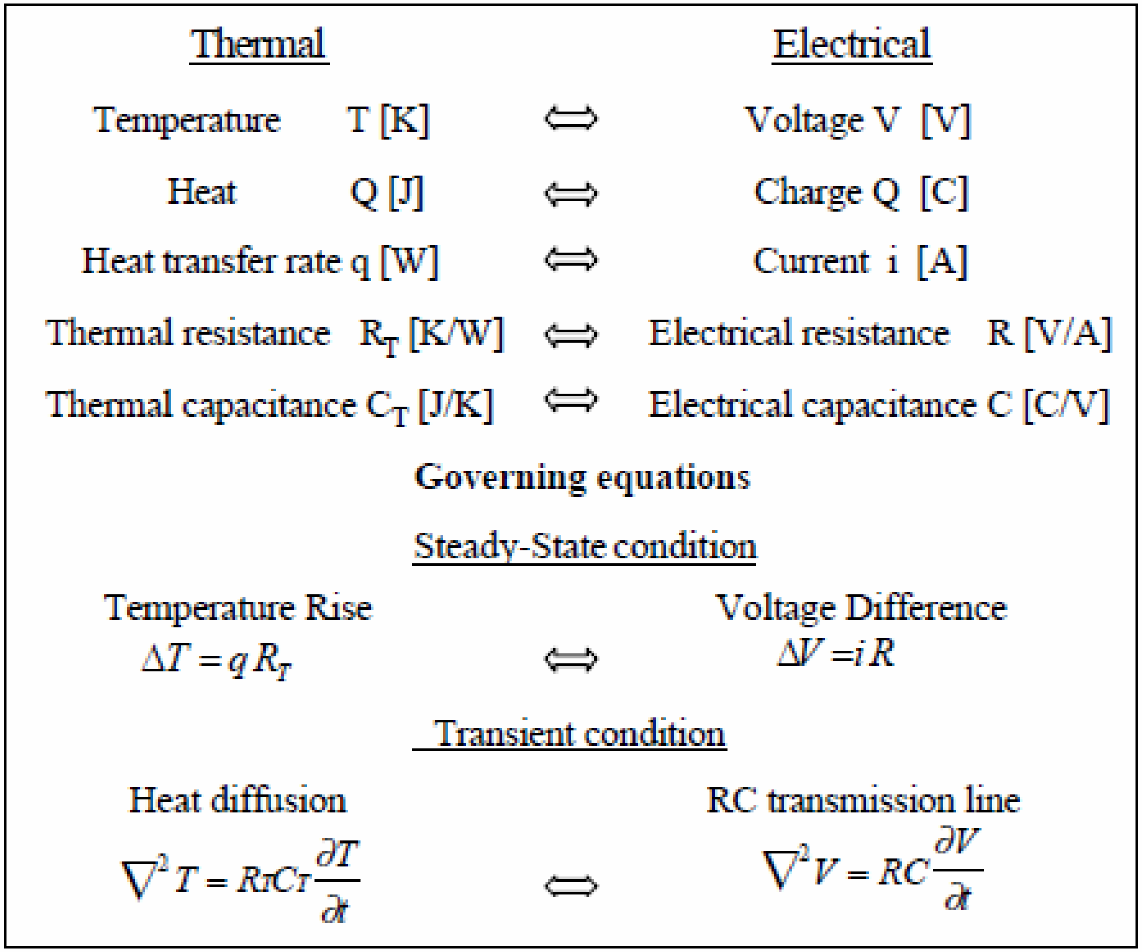
Electrical Engineering, Stanford University

Topics

1) Basics of Joule Heating
2) Heating in Devices & Circuits
3) **Thermal Resistance & Estimates**
4) Device Thermometry

Thermal-Electrical Cheat Sheet



Thermal		Electrical
Temperature T [K]	⇔	Voltage V [V]
Heat Q [J]	⇔	Charge Q [C]
Heat transfer rate q [W]	⇔	Current i [A]
Thermal resistance R_T [K/W]	⇔	Electrical resistance R [V/A]
Thermal capacitance C_T [J/K]	⇔	Electrical capacitance C [C/V]

Governing equations

Steady-State condition

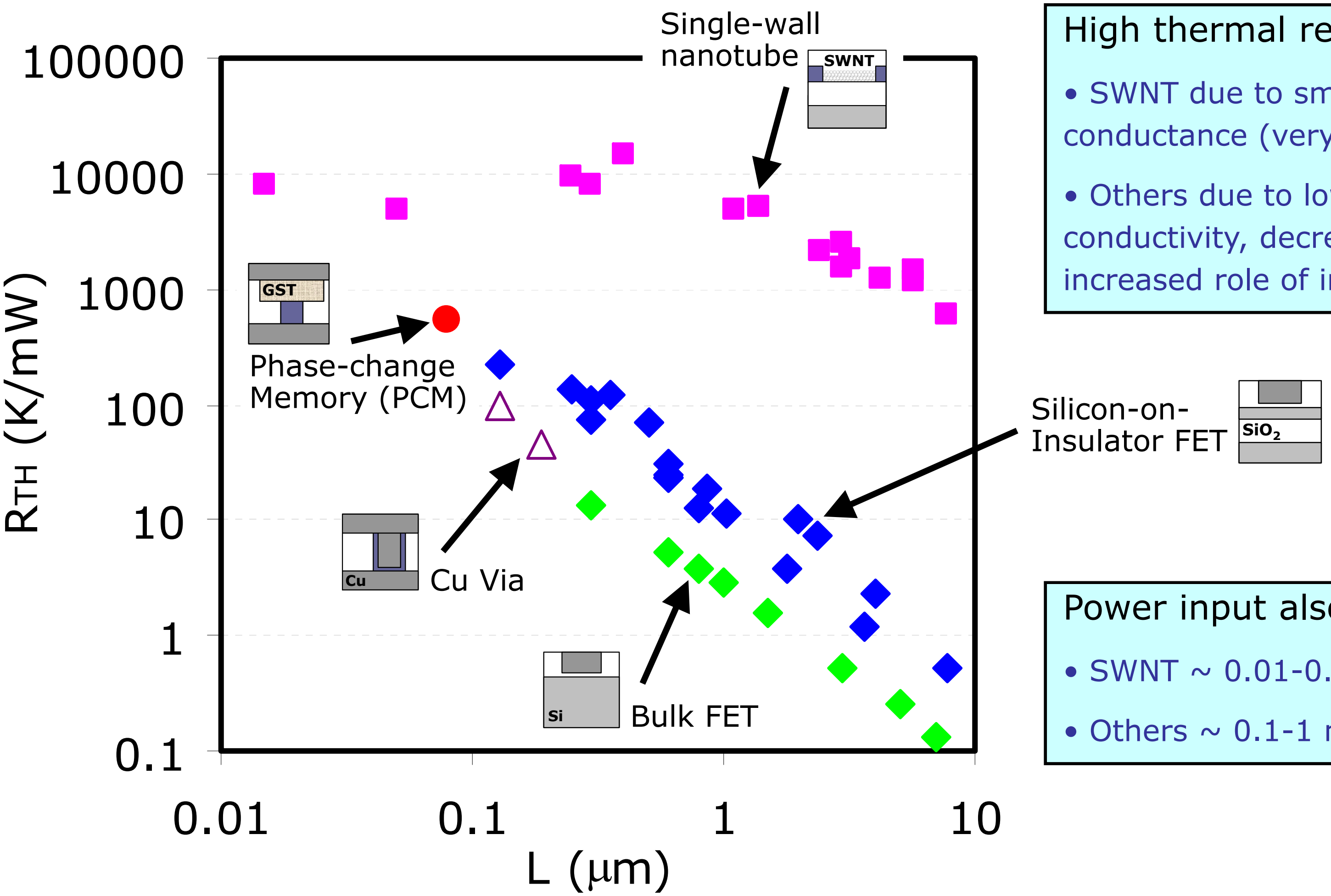
Temperature Rise $\Delta T = q R_T$	⇔	Voltage Difference $\Delta V = i R$

Transient condition

Heat diffusion $\nabla^2 T = R_T C_T \frac{\partial T}{\partial t}$	⇔	RC transmission line $\nabla^2 V = RC \frac{\partial V}{\partial t}$

Figure 1. Thermal-Electrical analogous quantities.

Device Thermal Resistance Data



High thermal resistances:

- SWNT due to small thermal conductance (very small d ~ 2 nm)
- Others due to low thermal conductivity, decreasing dimensions, increased role of interfaces

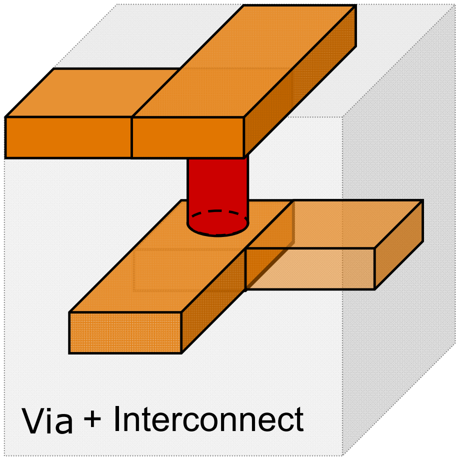
Power input also matters:

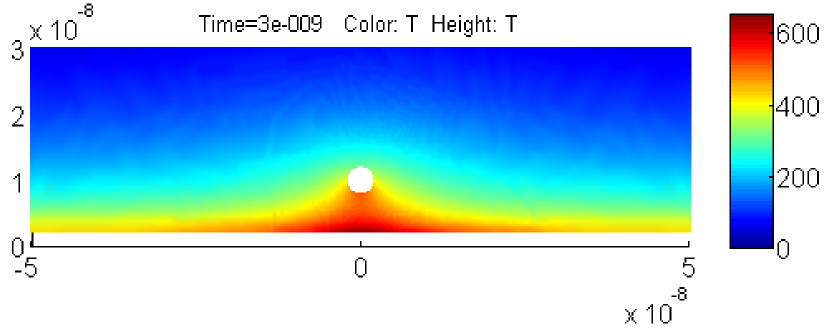
- SWNT ~ 0.01-0.1 mW
- Others ~ 0.1-1 mW

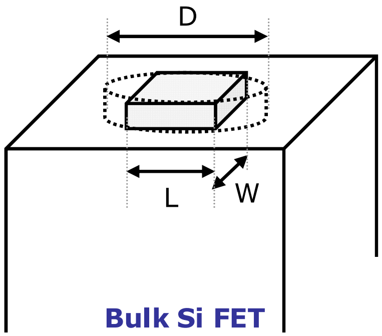
Data: Mautry (1990), Bunyan (1992), Su (1994), Lee (1995), Jenkins (1995), Tenbroek (1996), Jin (2001), Reyboz (2004), Javey (2004), Seidel (2004), Pop (2004-6), Maune (2006).

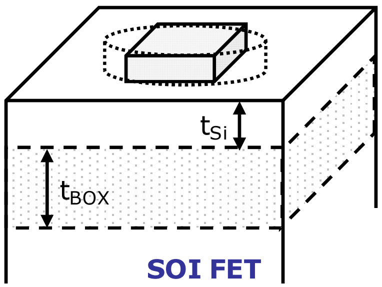
Approaches for Thermal Resistance

- Time scale:
  - **Steady-State (DC)**
  - Transient
- Geometric complexity:
  - Lumped element (shape factors)
  - Analytic
  - Finite element (Fourier law)



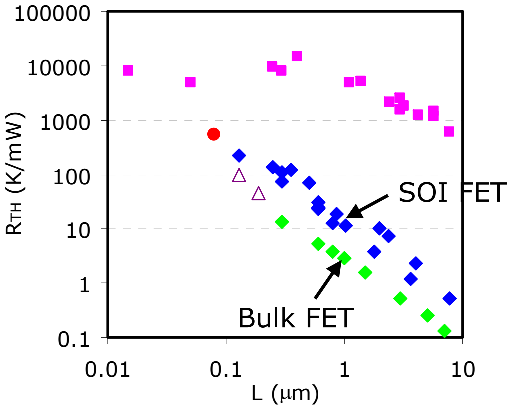


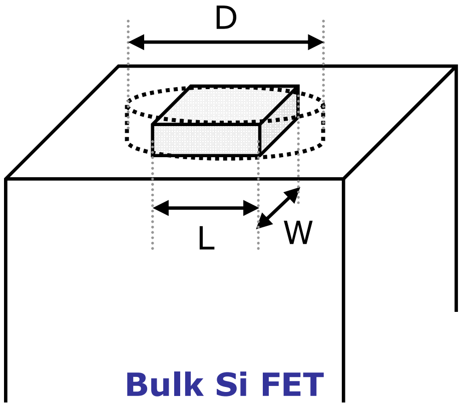


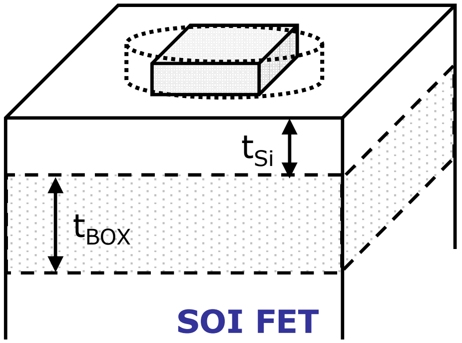


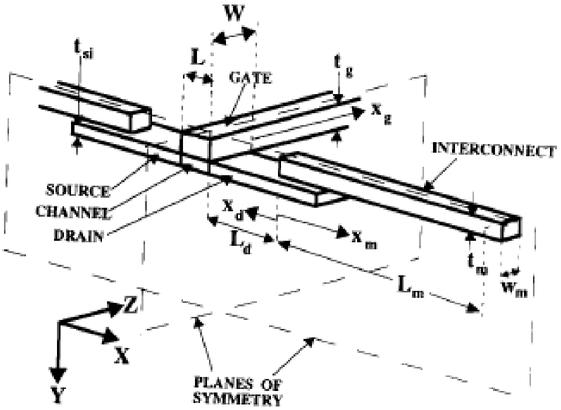
Modeling Device Thermal Resistance

- Steady-state (DC) models
  - Lumped: Mautry (1990), Goodson-Su (1994-5), Pop (2004), Darwish (2005)
  - Finite-Element





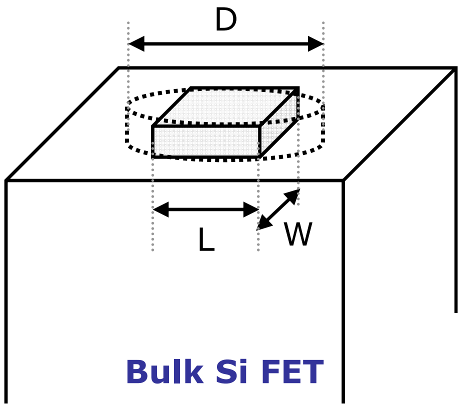




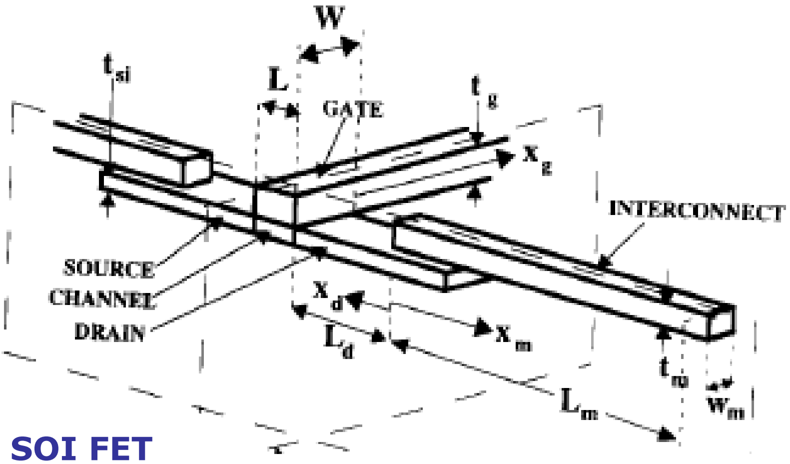
$$R_{TH} = \frac{1}{2k_{Si}D} \approx \frac{1}{2k_{Si}\sqrt{LW}} S$$

$$R_{TH} \approx \frac{1}{2W}\left(\frac{t_{BOX}}{k_{BOX}k_{Si}t_{Si}}\right)^{1/2}$$

Examples (1)



Bulk Si FET



SOI FET

$$R_{TH} = \frac{1}{2k_{Si}D} \approx \frac{1}{2k_{Si}\sqrt{LW}}S$$

k_{Si} ~ 100 Wm^{-1}K^{-1} (highly doped Si)
and "D" = 1 μm

then R_{TH} ~ 5 K/mW

so $\Delta T = PR_{TH}$ ~ 5 K with 1 mW power

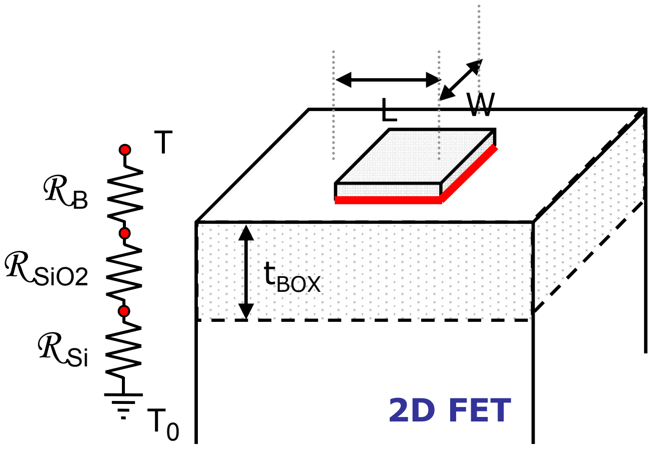
$$R_{TH} \approx \frac{1}{2W}\left(\frac{t_{BOX}}{k_{BOX}k_{Si}t_{Si}}\right)^{1/2}$$

t_{Si} = 10 nm, t_{BOX} = 50 nm, W = 1 μm
k_{Si} ~ 10 Wm^{-1}K^{-1} (reduced in thin film)
and k_{BOX} ~ 1.4 Wm^{-1}K^{-1}

then R_{TH} ~ 130 K/mW

so $\Delta T = PR_{TH}$ ~ 67 K with 0.5 mW power

Examples (2)



t_{BOX} = 90 nm, L = W = 1 µm and $k_{BOX} \sim 1.4$ W/m/K
TBR = thermal boundary resistance ~ 10^{-8} m^2K/W

$\mathcal{R}_B$ = TBR / (WL) ~ 10 K/mW

$\mathcal{R}_{SiO2} = t_{BOX} / (k_{BOX} * WL)$ ~ 60 K/mW

$\mathcal{R}_{Si} = 1 / [2 * k_{Si} * (W + t_{BOX})]$ ~ 4.5 K/mW

total R_{TH} ~ 75 K/mW so $\Delta T = PR_{TH}$ ~ 30 K with 0.4 mW power

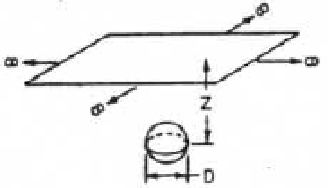
note the SiO_2 layer dominates, but TBR also plays an important role

Shape Factors

Sunderland, ASHRAE (1964), many others

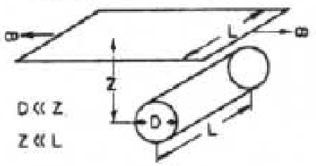
- Heat flux: $q = Sk(T_1 - T_0)$
- Equivalent thermal resistance $R_{TH} = 1/Sk$

1. Sphere to Infinite Plane



$$S = \frac{2\pi D}{1 + (D/4Z)}$$

2. Cylinder of Length L Parallel to Plane of Infinite Width and Length L

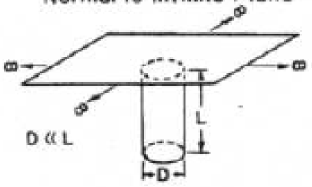


$$S \approx \frac{2\pi L}{\ln(4Z/D)}$$

a more exact solution is :

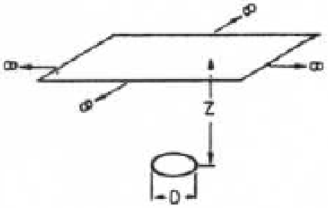
$$S = \frac{2\pi L}{\cosh^{-1}(2Z/D)}$$

3. Circular Cylinder of Length L Normal to Infinite Plane



$$S = \frac{2\pi L}{\ln(4L/D)}$$

4. Thin Circular Disk Parallel to and Well Below Infinite Plane

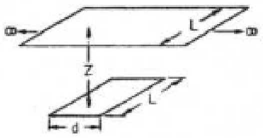


$S \approx 4D$

more exact :

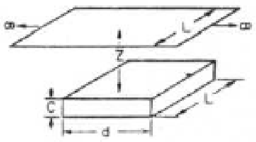
$$S = \frac{4.44D}{1 - (D/5.66Z)}$$

5. Plate of Width d and Length L Parallel to Plane of Infinite Width and Length L.

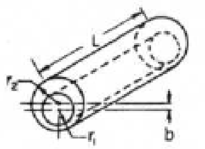


$$S = 1.45\left[\log\frac{Z+d}{d}\right]^{-0.59} L \quad 4$$

6. Rectangular Prism of Length L Parallel to Plane of Infinite Width and Length L.

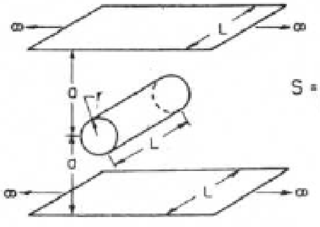


$$S = 1.685\left[\log\frac{Z+d}{d}\right]^{-0.59}\left[\frac{Z}{c}\right]^{-0.078} L \quad 4$$



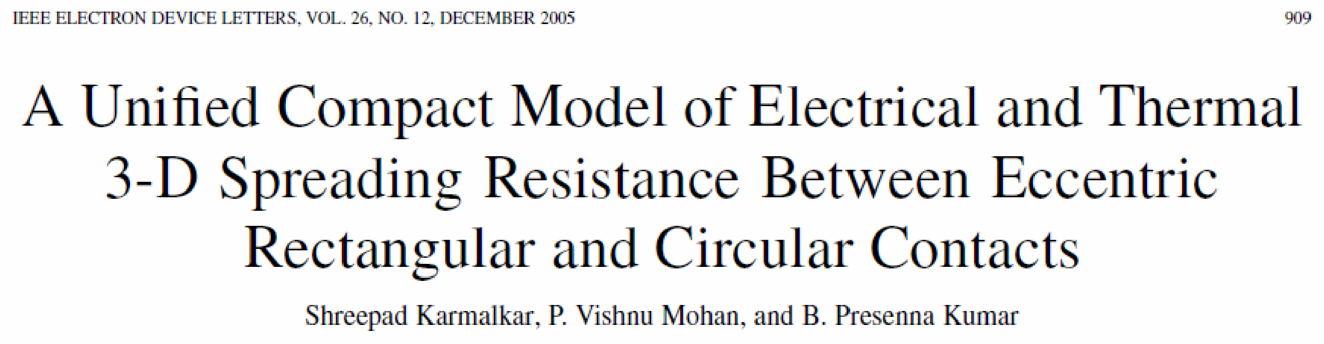
$$S = \frac{2\pi L}{\cosh^{-1}\left[\frac{r_1^2 + r_2^2 - b^2}{2 r_1 r_2}\right]}$$

12. Cylinder of Length L between Parallel Planes of Infinite Width and Length L.



$$S = \frac{2\pi L}{\ln(4a/\pi r)}$$

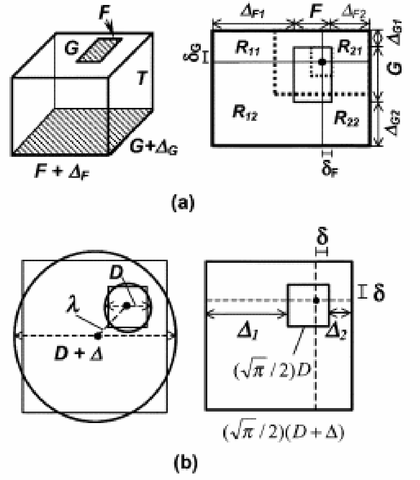
Many Shape Factors (Compact Models)



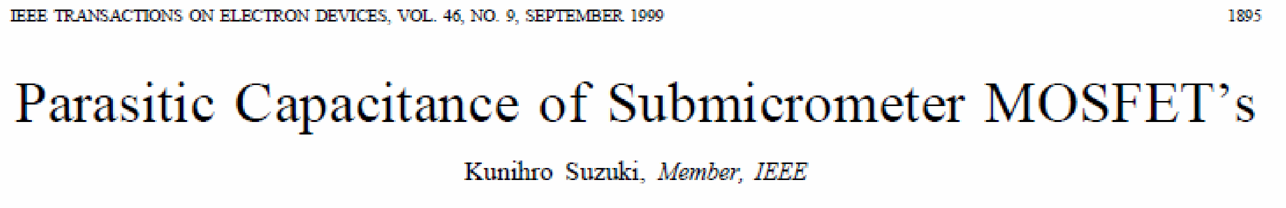
IEEE ELECTRON DEVICE LETTERS, VOL. 26, NO. 12, DECEMBER 2005 909

A Unified Compact Model of Electrical and Thermal 3-D Spreading Resistance Between Eccentric Rectangular and Circular Contacts

Shreepad Karmalkar, P. Vishnu Mohan, and B. Presenna Kumar



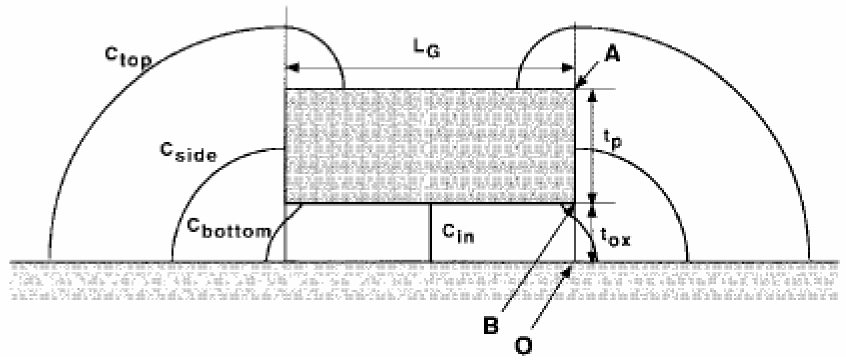
$$\frac{R^{\Delta_F=0}_{\Delta_G\to\infty}}{R^{\Delta_F=0}_{\Delta_G=0}} = \frac{G}{2T\tan\alpha_G}\ln\left(1+\frac{2T\tan\alpha_G}{G}\right)$$



IEEE TRANSACTIONS ON ELECTRON DEVICES, VOL. 46, NO. 9, SEPTEMBER 1999 1895

Parasitic Capacitance of Submicrometer MOSFET's

Kunihro Suzuki, *Member, IEEE*



$$C_{\text{side}} = \frac{\varepsilon_{ox}}{\pi}\ln[a]$$

$$a = 2K(K^2-1)^{1/2} + 2K - 1$$
$$K = 1 + \frac{t_p}{t_{ox}}$$

Obtaining the Temperature Distribution

- So far we've only looked at lumped thermal models
- Now we want temperature <u>distribution</u> T(x)
- Simplest case: Si layer on SiO_2/Si substrate (SOI)
- Or interconnect on thermally insulating SiO_2

$$T_m - T_0 = Z_1 \cosh[m_m(L_m - x_m)] \qquad (1)$$

$$T_d - T_0 = Z_2 \exp[m_d x_d] + Z_3 \exp[-m_d x_d] \qquad (2)$$

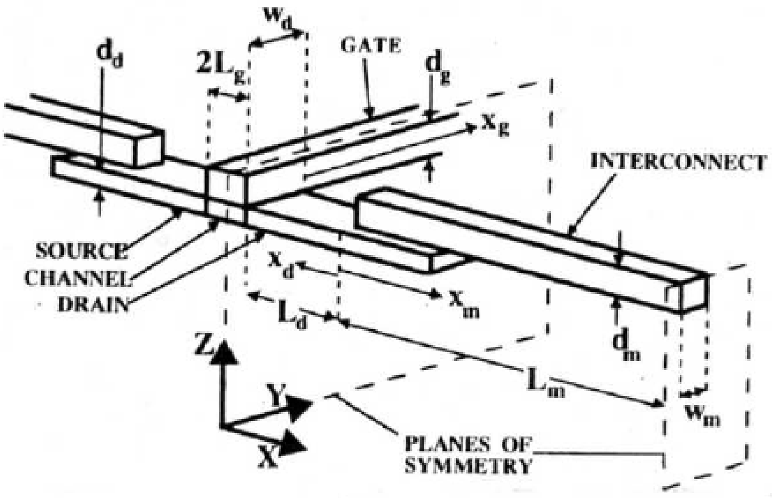
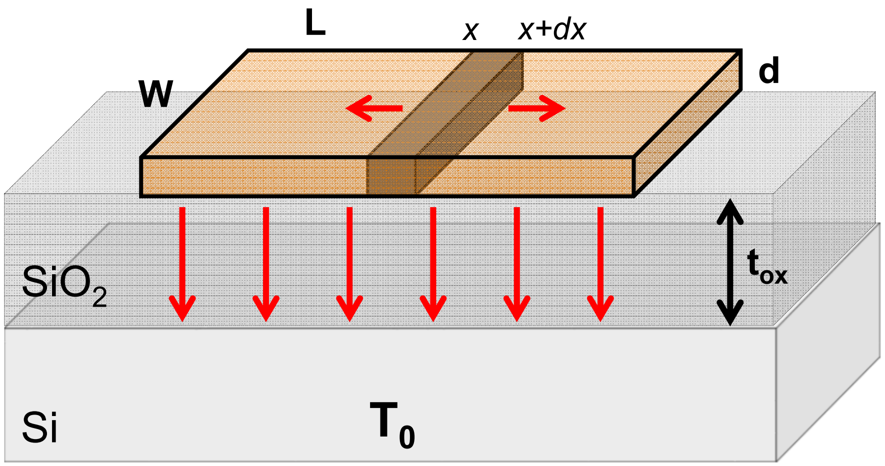


Fig. 2 Geometry of the thermal model of a SOI FET (Goodson and Flik, 1992)

1-D Interconnect with Heat Generation



Heat: $Q = -Ak\frac{dT}{dx}$

Electrical: $I = AJ = A(\sigma F) = A\sigma\frac{dV}{dx}$

Write energy balance equation
for element “dx”
pick units of J or W (= J/s)

<u>Energy In</u> (here, Joule heat) = <u>Energy Out</u> (left, right, bottom) + <u>Change in Internal Energy</u>

$$Q'''Adx \qquad -kA\left.\frac{\partial T}{\partial x}\right|_{x} - kA\left.\frac{\partial T}{\partial x}\right|_{x+dx} + hWdx\left(T - T_0\right) \qquad C\left(Adx\right)\frac{\partial T}{\partial t}$$

divide by (Adx):

$$\boxed{Q''' + \nabla\left(k\nabla T\right) - h\frac{W}{A}\left(T - T_0\right) = C\frac{\partial T}{\partial t}}$$

e.g. $J \cdot E$ if non-uniform
or $I^2R/(WLd)$ if uniform

convection-like term, here $h = k_{ox}/t_{ox}$ and $W/A = 1/d$
“W” can be “perimeter” if heat loss in all directions

Ex: 1D Rectangular Nanowire

JOURNAL OF APPLIED PHYSICS VOLUME 86, NUMBER 3 1 AUGUST 1999

Analysis of failure mechanisms in electrically stressed Au nanowires

C. Durkan,[a] M. A. Schneider, and M. E. Welland
Department of Engineering, Nanoscale Science Group, University of Cambridge, Trumpington Street, Cambridge CB2 1PZ, United Kingdom

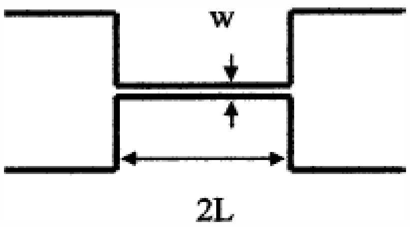


FIG. 1. Schematic of wire geometry.

$$\nabla^2 T - m^2 T + \frac{Q}{k} = 0, \qquad (1)$$

where

$$m = \sqrt{\frac{k_{\mathrm{sub}}}{ktd}},$$

$1/m$ is natural length scale
"thermal healing length"

$$\boxed{L_H = \sqrt{\frac{k}{k_{sub}} t d_{sub}}}$$

where $Q = J^2\rho$, k, k_{sub}, t and d are the thermal conductivity and thickness of the wire and the substrate, respectively. It should be noted that the resistivity of such a film will be approximately 2.5 times larger than the bulk value due to grain boundary scattering and the Fuchs size effect.[5] The

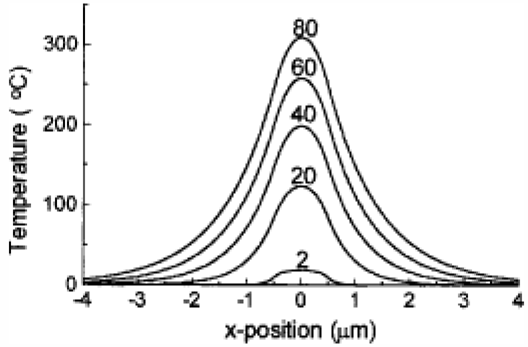
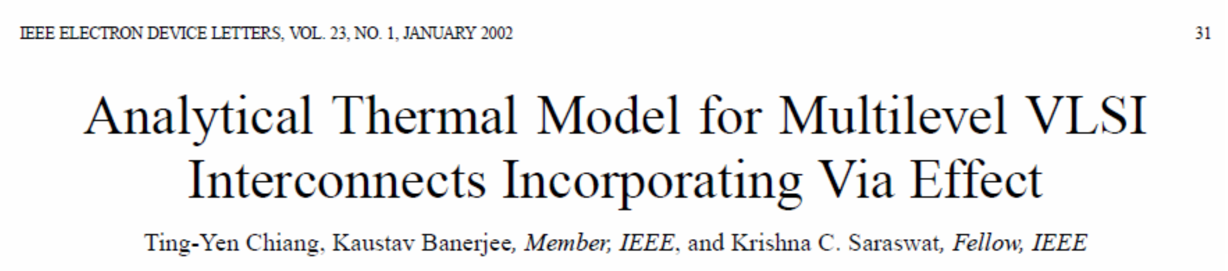


FIG. 2. Calculated temperature along 20 nm thick and 1000 nm long Au wires carrying 2×10^{12} A m^{-2}, for several substrate oxide thicknesses (in nm).

$$T_{\mathrm{wire}} = -\frac{Q}{2km^2} e^{-mL}(e^{mx} + e^{-mx}) + Qe^{-mx}\left(\frac{td}{k_{\mathrm{sub}}} - \frac{1}{km^2}\right) + \frac{Q}{km^2}, \qquad (2)$$

$$T_{\mathrm{contact}} = Q\left[\frac{td}{k_{\mathrm{sub}}} - \frac{1}{km^2} + \frac{1}{2km^2}(e^{mL} - e^{-mL})\right] e^{-m|x|}. \qquad (3)$$

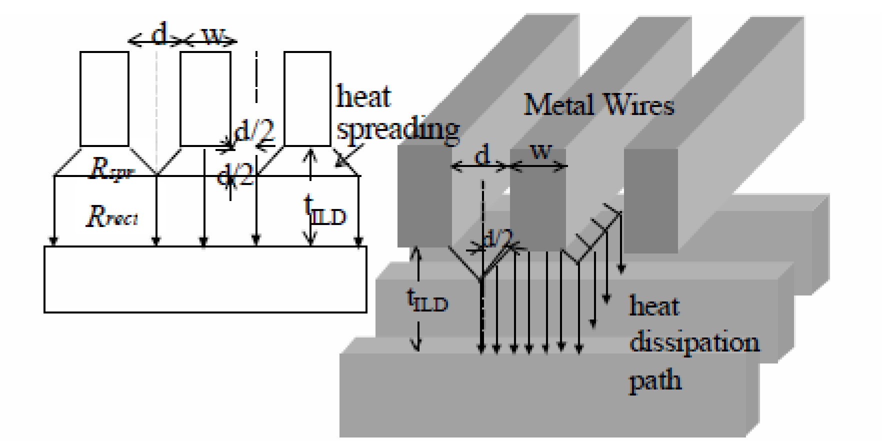
Interconnect Heat Loss and Crosstalk

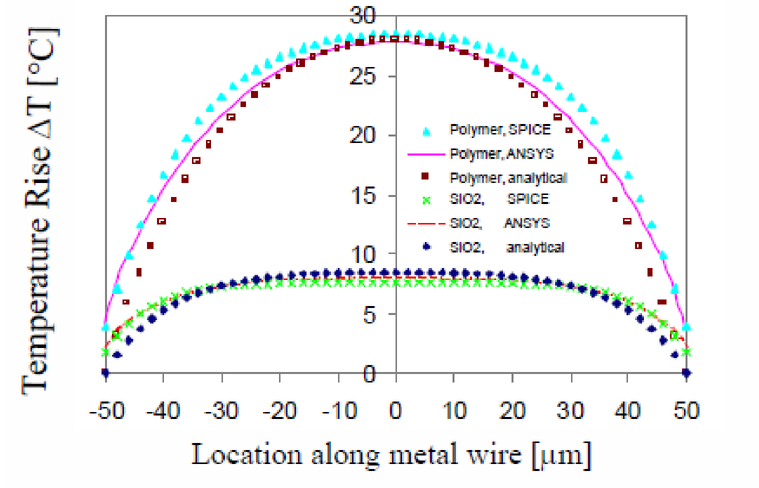


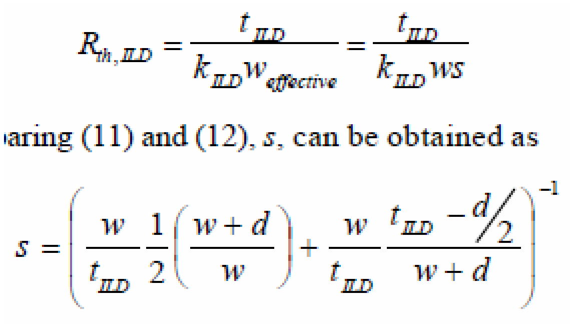
IEEE ELECTRON DEVICE LETTERS, VOL. 23, NO. 1, JANUARY 2002 31

Analytical Thermal Model for Multilevel VLSI Interconnects Incorporating Via Effect

Ting-Yen Chiang, Kaustav Banerjee, *Member, IEEE*, and Krishna C. Saraswat, *Fellow, IEEE*



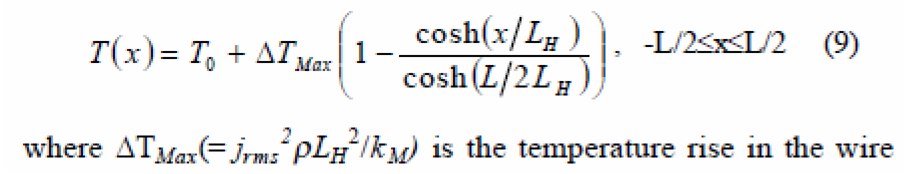




$$R_{th,ILD} = \frac{t_{ILD}}{k_{ILD} w_{effective}} = \frac{t_{ILD}}{k_{ILD} w s}$$

aring (11) and (12), s, can be obtained as

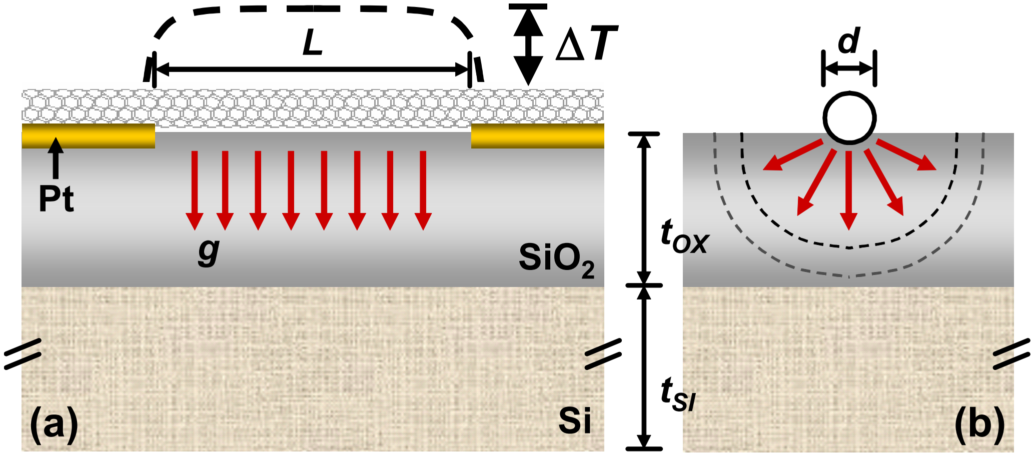
$$s = \left(\frac{w}{t_{ILD}} \frac{1}{2} \left(\frac{w+d}{w} \right) + \frac{w}{t_{ILD}} \frac{t_{ILD} - d/2}{w+d} \right)^{-1}$$



$$T(x) = T_0 + \Delta T_{Max} \left(1 - \frac{\cosh(x/L_H)}{\cosh(L/2L_H)} \right), \quad -L/2 \le x \le L/2 \qquad (9)$$

where $\Delta T_{Max} (= j_{rms}^2 \rho L_H^2 / k_M)$ is the temperature rise in the wire

Ex: Carbon Nanotube (Cylinder)

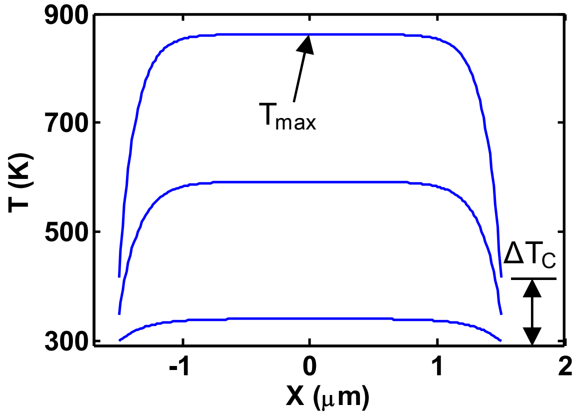


$$A\nabla(k\nabla T) + p' - g(T - T_0) = 0$$

$$\boxed{T(x) = T_0 + \frac{p'}{g}\left[1 - \frac{\cosh(x/L_H)}{\cosh(L/2L_H)}\right]}$$

$$p' = I^2\frac{dR}{dx} = I^2\frac{h}{4q^2}\frac{1}{\lambda_{eff}} \qquad L_H = \sqrt{\frac{kA}{g}}$$

Role of cylindrical heat spreading (shape factor!) $\Big\}$ $g_{ox} = \dfrac{\pi k_{ox}}{\ln\left(\dfrac{8t_{ox}}{\pi d}\right)}$



Role of thermal contact resistance

$$\left\{\; kA\frac{dT}{dx}\bigg|_C = \frac{\Delta T_C}{\mathcal{R}_{C,Th}}\right.$$

E. Pop *et al.*
J. Appl. Phys. 101, 093710 (2007)

Further Reading

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